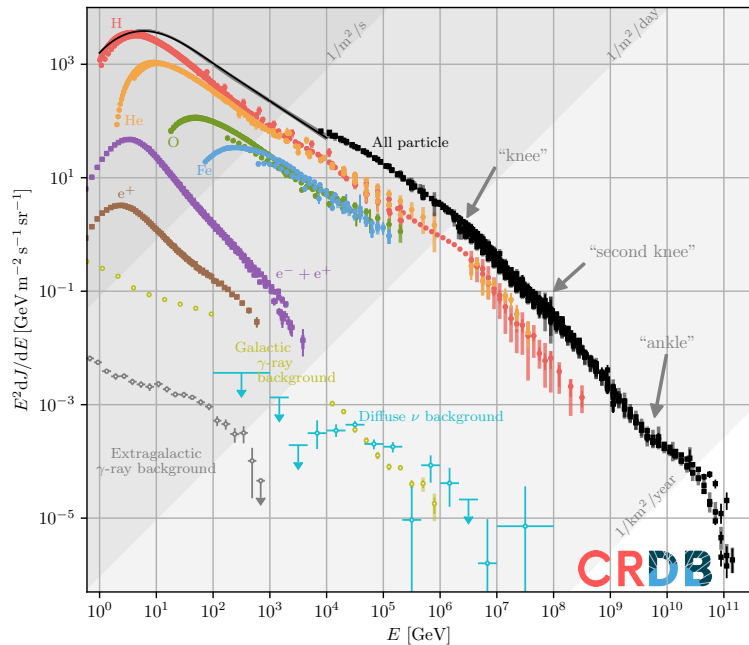


GeV cosmic rays: measurements and interpretation

Philipp Mertsch

CDHY lecture series on the extreme non-thermal universe
4 March 2026

The cosmic ray spectrum

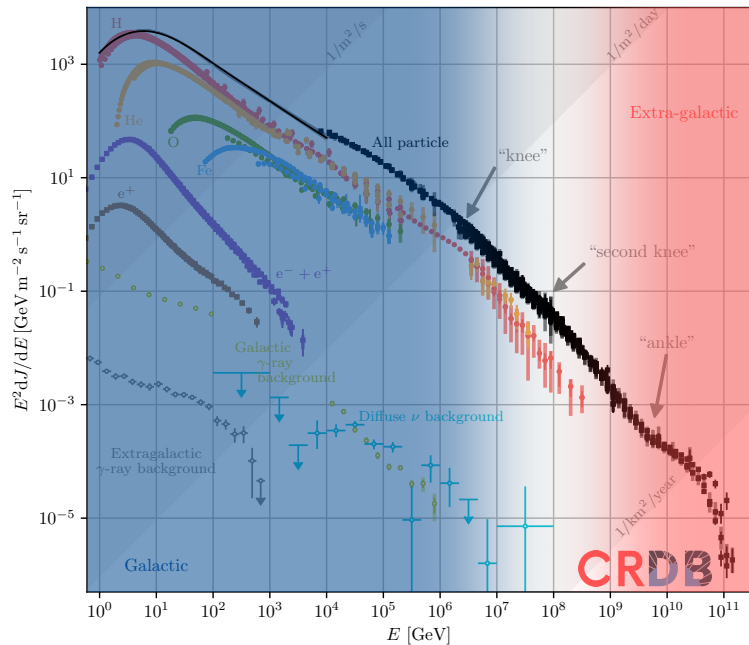


Intensity dJ/dE

$$\frac{dJ}{dE} \equiv \frac{(\# \text{particles})}{\Delta t \Delta A \Delta \Omega \Delta E}$$

- ~ 12 orders of magnitude in energy
- $\sim$ power law $dJ/dE \propto E^{-3}$ with some features

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Why are cosmic rays interesting?

Cosmic ray origin

- What are the sources of cosmic rays?
- Century-old problem!
- Astrophysical interest

Ingredient

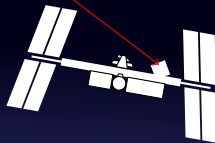
Cosmic rays ...

- produce diffuse emission
- ionise and heat
- provide gravitational support
- drive winds
- generate turbulence

Exotic probe

- Indirect searches of dark matter
- Dark matter cools cosmic rays, cosmic rays scatter dark matter
Bringmann et al. (2019), Ng et al. (2019)
- Primordial anti-matter?

Space experiments: AMS-02, CALET, DAMPE, *Fermi*-LAT

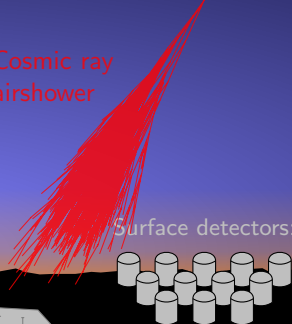


Balloon experiments:



CREAM
HELIX
GAPS

Cosmic ray
airshower



Cherenkov telescopes:
HESS, VERITAS, MAGIC



Fluorescence detectors:

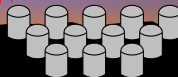
Auger, TA



IceCube



Surface detectors:



HAWC, IceTop, Auger, TA, LHAASO

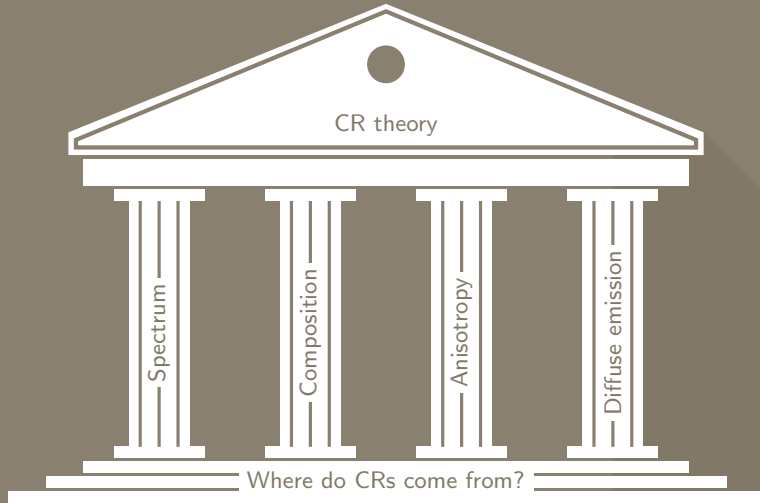
Outline

- ① Introduction
- ② Fundamental observations and key insights
- ③ Standard picture of cosmic rays
- ④ Measurements and interpretation 1: Spectral breaks
- ⑤ Measurements and interpretation 2: Non-universality
- ⑥ Measurements and interpretation 3: Spectral coincidences
- ⑦ Measurements and interpretation 4: Electrons and positrons
- ⑧ Summary

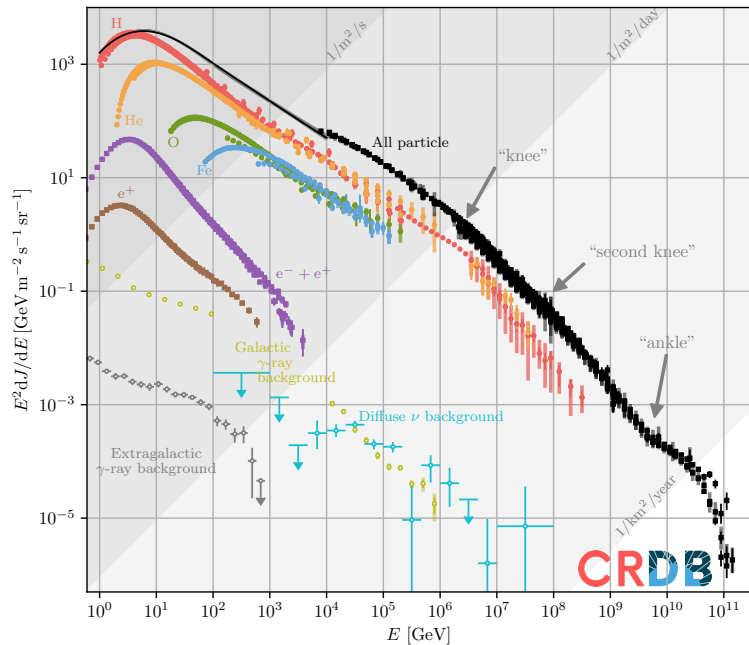
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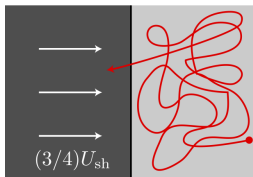
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A gambling analogy

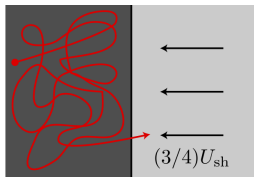
- Amount p ; probability of not having lost N
- Chance of losing q : $N_{i+1} = N_i(1 - q)$
- Fractional gain g : $p_{i+1} = p_i(1 + g)$
- Probability of having more than p_n : $N(> p_n)$

$$\left. \begin{aligned} \ln N(> p_n)/N_0 &= \ln(1 - q)^n \simeq -nq \\ \ln p_n/p_0 &= \ln(1 + g)^n \simeq ng \end{aligned} \right\} \Rightarrow N(> p_n) = N_0 \left(\frac{p_n}{p_0} \right)^{-q/g} = \int_{p_0}^{p_n} dp \frac{dN}{dp}$$

$$\frac{dN}{dp} \propto \left(\frac{p}{p_0} \right)^{-(1+q/g)}$$



upstream rest frame

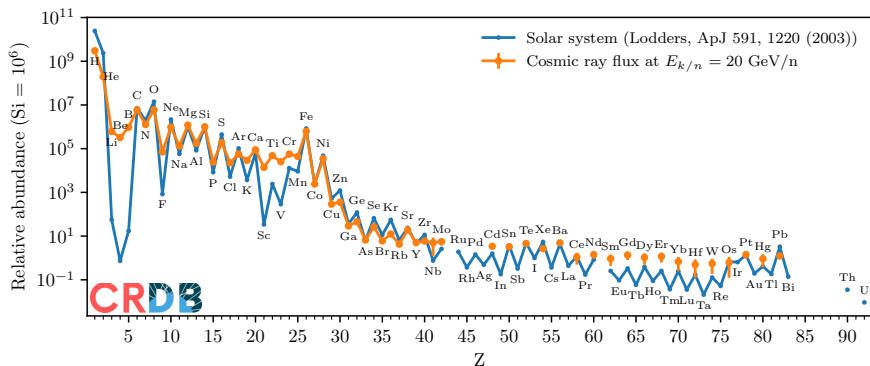


downstream rest frame

Diffusive shock acceleration

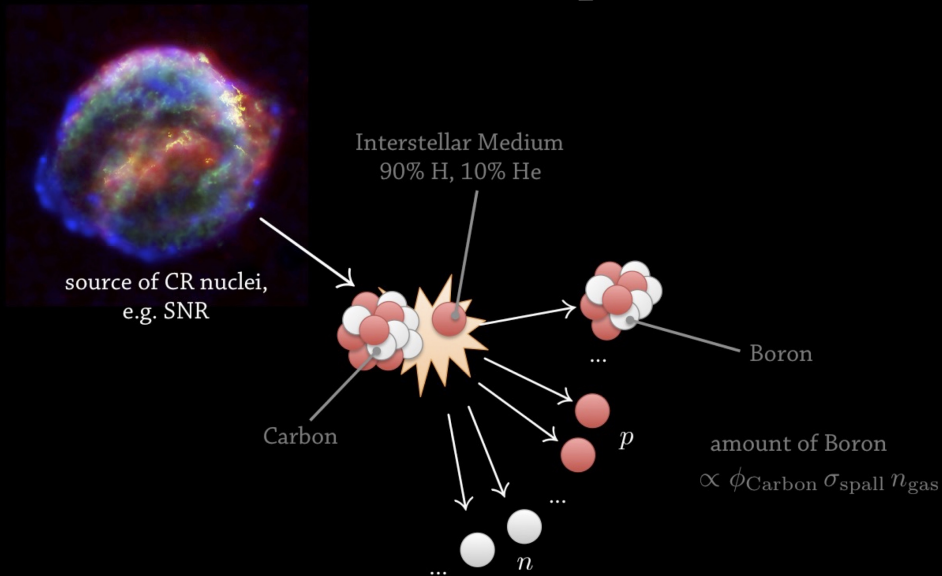
- Prob. of escape: $q = \frac{U_{sh}}{v}$
- fractional gain: $g = \frac{\Delta p}{p} = \frac{U_{sh}}{v}$
- spectrum: $\frac{dN}{dp} \propto p^{-2}$

- Some species have same abundances in CRs and in solar system → **primaries**
- Other species are overabundant with respect to solar abundances:

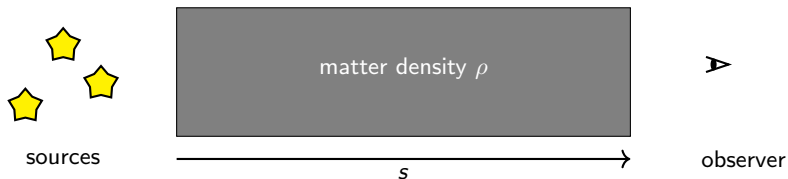


→ Must have been produced during the transport → **secondaries**

Secondaries from spallation



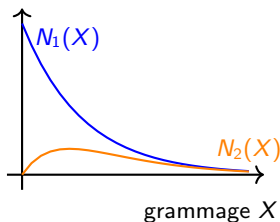
Slab model



- Define CR grammage:
$$X \equiv \int ds' \rho(s') = s\bar{\rho}$$
- Consider number of primary and secondary CRs, N_1 and N_2 :

$$\frac{dN_1}{dX} = -\frac{N_1}{\lambda_1}$$

$$\frac{dN_2}{dX} = -\frac{N_2}{\lambda_2} + \text{BR}_{1 \rightarrow 2} \frac{N_1}{\lambda_1}$$



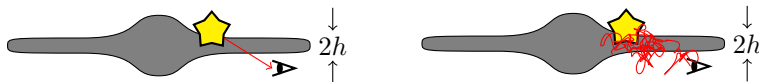
with $1/\lambda_{1,2} = \sigma_{1,2}/m$ the specific cross-section

- Observations: $N_2/N_1 \simeq 0.3 \rightarrow X \simeq 7.2 \text{ g cm}^{-2}$

- Where does the grammage come from?
- If CRs traverse the Galactic disk, every crossing contributes

$$\Delta X \sim h m_N n_{\text{gas}} \simeq (100 \text{ pc})(1.7 \times 10^{-24} \text{ g})(1 \text{ cm}^{-3}) \simeq 5 \times 10^{-4} \text{ g cm}^{-2}$$

- ($1 \text{ pc} \simeq 3.1 \times 10^{18} \text{ cm}$)



CRs must cross the disk many times, e.g. through **diffusion**

- Residence time in disk:

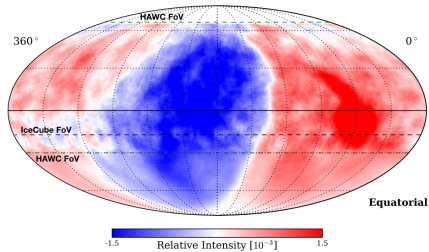
$$t_{\text{esc}} = \frac{s}{v} = \frac{X}{v \bar{\rho}} = \frac{X}{v m_N \bar{n}_{\text{gas}}} \simeq 3 \times 10^6 \text{ yr}$$

for $n_{\text{gas}} = 1 \text{ cm}^{-3}$

Anisotropy (I)

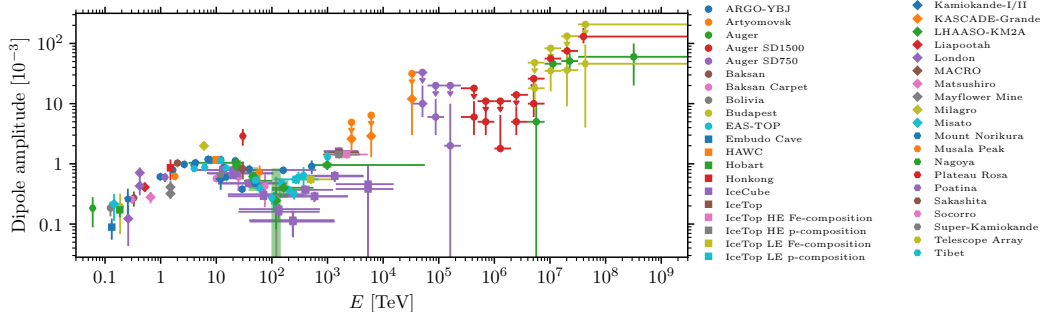
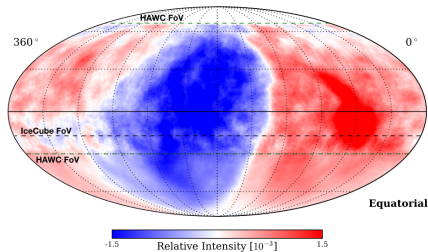
- Angular distribution of CRs is very isotropic

- E.g., the dipole anisotropy $a \equiv \frac{\phi_{\max} - \phi_{\min}}{\phi_{\max} + \phi_{\min}}$



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- Between a few GeV and a PeV: $a = \mathcal{O}(10^{-4} \dots 10^{-3})$



Anisotropy (II)

Between a few GeV and a PeV: $a = \mathcal{O}(10^{-4} \dots 10^{-3})$



- Sources are discrete
- If CRs were travelling ballistically, would expect $\mathcal{O}(1)$ anisotropy
- See, e.g., electro-magnetic radiation

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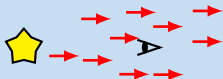
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- Need to isotropise CRs
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- Need to isotropise CRs
- (Coulomb) collisions with interstellar matter too infrequent

- Scattering of charged particles with turbulent magnetic field isotropises particle directions
- Particles perform a random walk in space:

$$\langle (\Delta r)^2 \rangle \propto \Delta t$$

- The constant of proportionality is called the **diffusion coefficient** κ

Outline

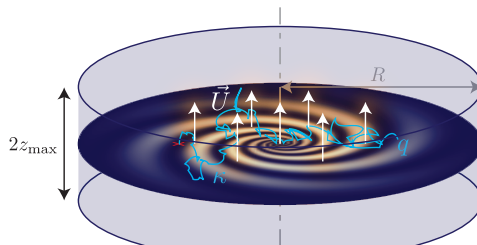
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The transport equation

$$\begin{aligned}\frac{\partial \psi_j}{\partial t} = & \nabla \cdot (\kappa \cdot \nabla \psi_j - \mathbf{U} \psi_j) && \text{spatial diffusion and advection} \\ & + \frac{\partial}{\partial p} \left(p^2 D_{pp} \frac{\partial}{\partial p} \frac{1}{p^2} \psi_j \right) && \text{momentum diffusion} \\ & + \frac{\partial}{\partial p} \left(- \frac{d\mathbf{p}}{dt} \psi_j + \frac{p}{3} (\nabla \cdot \mathbf{U}) \psi_j \right) && \text{momentum change incl. adiabatic} \\ & - v n_{\text{gas}} \sigma_j \psi_j - \frac{\psi_j}{\tau_j} && \text{spallation and decay} \\ & + v n_{\text{gas}} \sum_{k>j} \sigma_{k \rightarrow j} \psi_k + \sum_{k>j} \frac{\psi_k}{\tau_{k \rightarrow j}} && \text{spallation and decay} \\ & + q_j && \text{primary sources}\end{aligned}$$

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Simplified 1D model

- 1D approximation; homogeneous diffusion, advection

$$\frac{\partial \psi_j}{\partial t} - \kappa \frac{\partial^2 \psi_j}{\partial z^2} - U \frac{\partial \psi_j}{\partial z} + \left(v n_{\text{gas}}(z) \sigma_j + \frac{1}{\tau_j} \right) \psi_j = q_j(z) + \sum_{j < k} \left(v n_{\text{gas}}(z) \sigma_{k \rightarrow j} + \frac{1}{\tau_{k \rightarrow j}} \right) \psi_k$$

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- Steady-state

• Infinitely thin disk of half-height h

$$-\kappa \frac{\partial^2 \psi_j}{\partial z^2} - U \frac{\partial \psi_j}{\partial z} + \left(2 \underbrace{h \delta(z) n_{\text{gas}}(0)}_{\text{green}} v \sigma_j + \frac{1}{\tau_j} \right) \psi_j = \underbrace{2 h \delta(z) q_j(0)}_{\text{purple}} + \sum_{j < k} \left(2 \underbrace{h \delta(z) n_{\text{gas}}(0)}_{\text{green}} v \sigma_{k \rightarrow j} + \frac{1}{\tau_{k \rightarrow j}} \right) \psi_k$$

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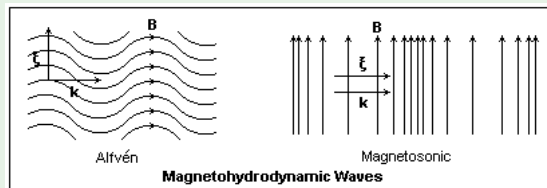
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Solution (without advection or decay)

$$\psi_j(0, \mathcal{R}) = \frac{h \left(q_j(0, \mathcal{R}) + \sum_{k > j} v n_{\text{gas}}(0) \sigma_{k \rightarrow j} \psi_k \right)}{\left(h v n_{\text{gas}}(0) \sigma_j + \frac{\kappa(\mathcal{R})}{z_{\text{max}}} \right)}$$

- CRs interact with the magnetic field in the background plasma
- Homogeneous field $\mathbf{B}_0 \rightarrow$ gyration
- Magneto-hydrodynamic (MHD) equations allow for small-amplitude solutions $\delta\mathbf{B}$: plasma waves

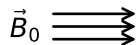
Examples



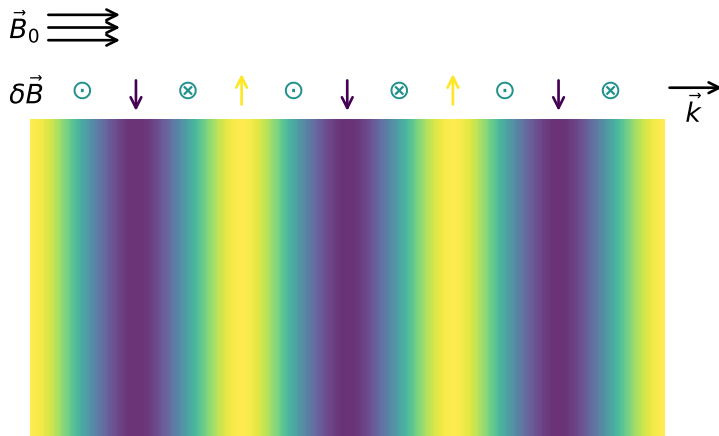
- Can perturbatively derive a diffusion equation:

$$\frac{\partial \psi}{\partial t} - \nabla \cdot (\kappa \cdot \nabla - \mathbf{U}) \psi = 0$$

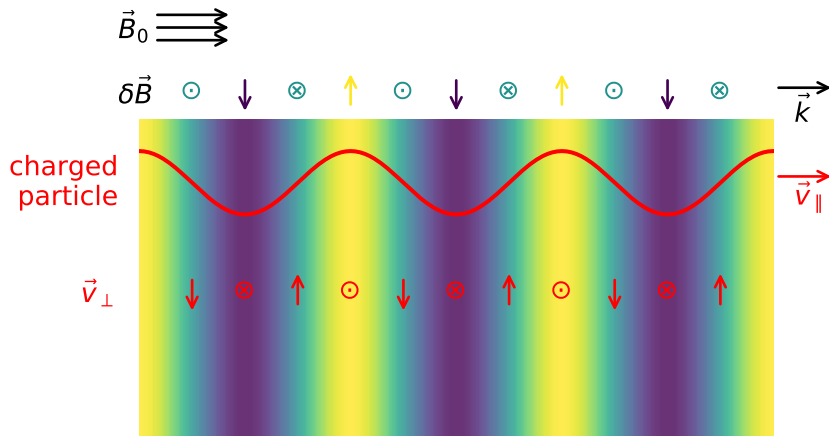
Momentum-dependence of diffusion coefficient



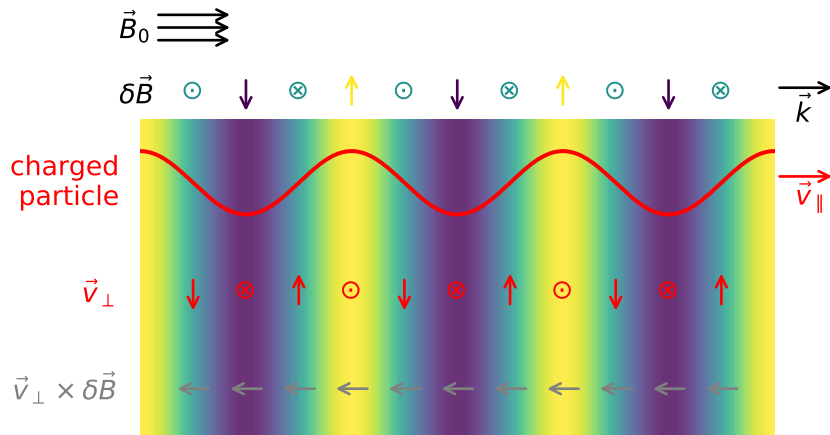
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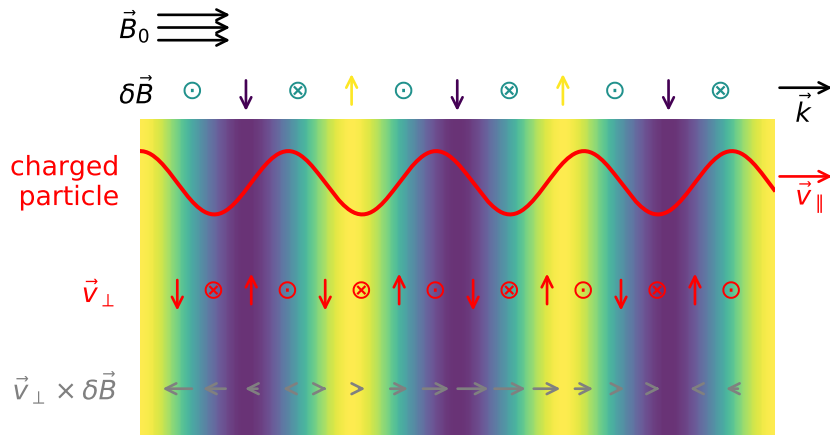


Momentum-dependence of diffusion coefficient



If $kr_g = 1 \Rightarrow$ coherent force over many gyrations

Momentum-dependence of diffusion coefficient

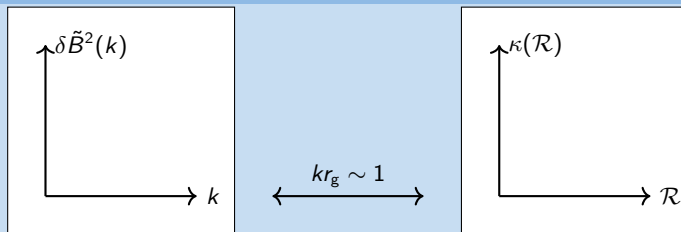


If $kr_g \neq 1 \Rightarrow$ **NO** coherent force

The diffusion coefficient

- Coherent force changes pitch-angle $\alpha = \arccos \frac{\mathbf{p} \cdot \mathbf{B}}{|\mathbf{p}| |\mathbf{B}|}$
- Repeated scattering with randomly phased waves $\rightarrow$ random walk in pitch-angle α
- Random changes of direction $\rightarrow$ spatial diffusion
- Rate depends on power spectrum of turbulent field

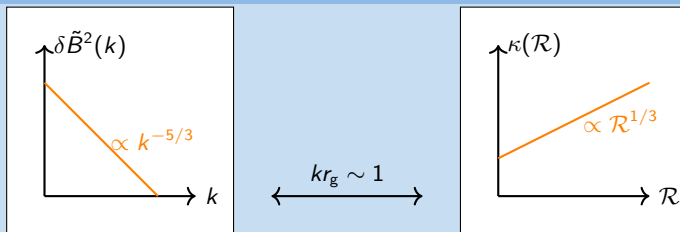
Resonant interactions



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Resonant interactions



- Charged particles interact resonantly with wave:

$$k_{\text{res}} = r_g^{-1} \propto p^{-1}$$

- Scattering rate Γ is gyrofrequency Ω reduced by density of resonant plasma waves:

$$\Gamma(k) \simeq \Omega \frac{k \delta \tilde{B}^2(k)}{B_0^2}$$

- The density of resonant plasma waves is:

$$\delta \tilde{B}^2(k) \sim \int d^3r e^{i\mathbf{r} \cdot \mathbf{k}} \langle \delta \mathbf{B}(\mathbf{r}_0) \delta \mathbf{B}(\mathbf{r}_0 + \mathbf{r}) \rangle \propto k^{-5/3}$$

- Interaction with ensemble of random waves $\rightarrow$ diffusion:

$$\kappa \simeq \frac{v^2}{3\Gamma} \propto \frac{1}{\Omega} \left(\frac{k \delta \tilde{B}^2(k)}{B_0^2} \right)^{-1} \Big|_{k=k_{\text{res}}} \propto p^{1/3}$$

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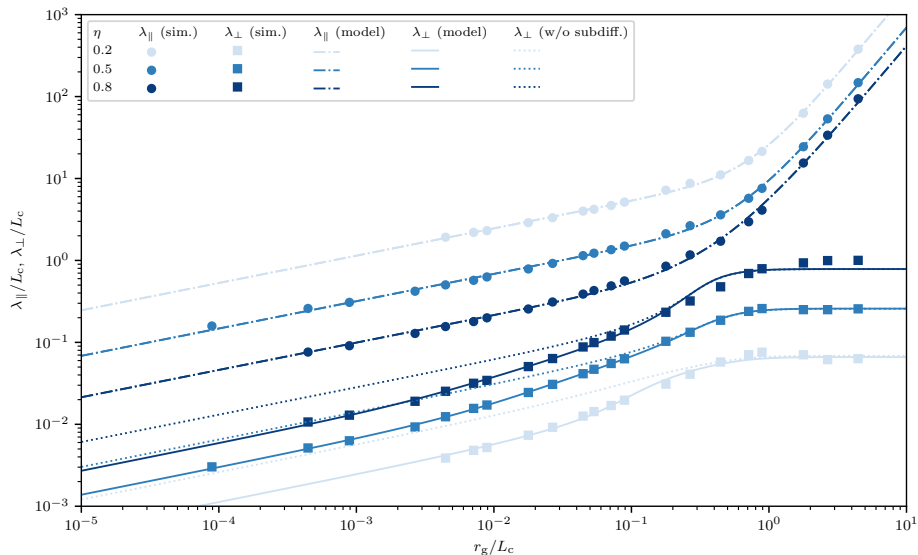
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Open questions

- Anisotropy of turbulence cascade?
- Importance of Alfvén vs magnetosonic?
- Non-wave-like perturbations?

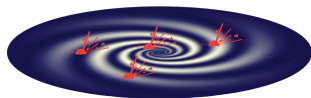
Mean-free paths

Kuhlen, Mertsch, Phan (2025)

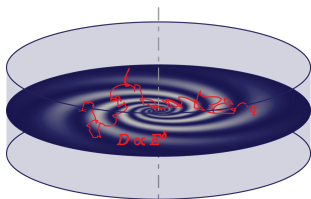


Cosmic ray *tenets*

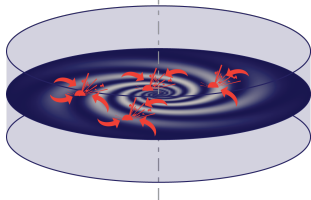
Ginzburg & Syrovatskii (1964)



- 1 Cosmic rays are accelerated in supernova remnants in the disk



- 2 Cosmic rays are diffusively confined to an extended halo



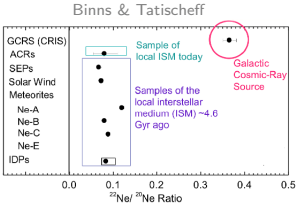
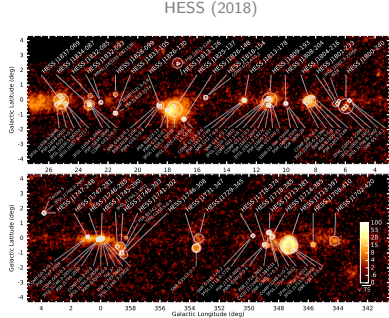
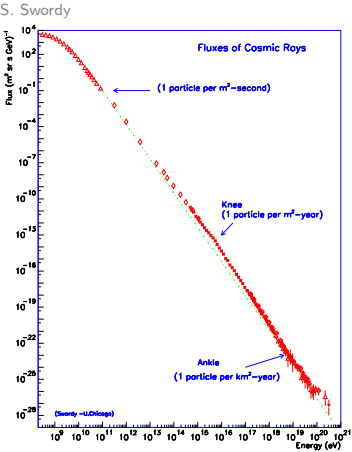
- 3 Supernova remnants accelerate material from the interstellar medium

Predictions in the standard picture

Universal power law spectra
Featureless power law below E_{knee}

PeVatrons = supernova remnants
Not observed yet?

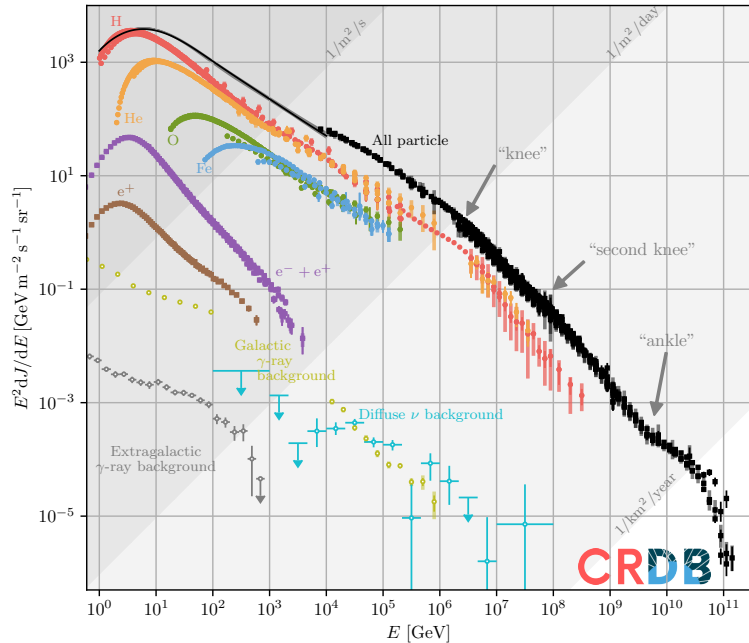
Composition
Solar abundances modulo spallation



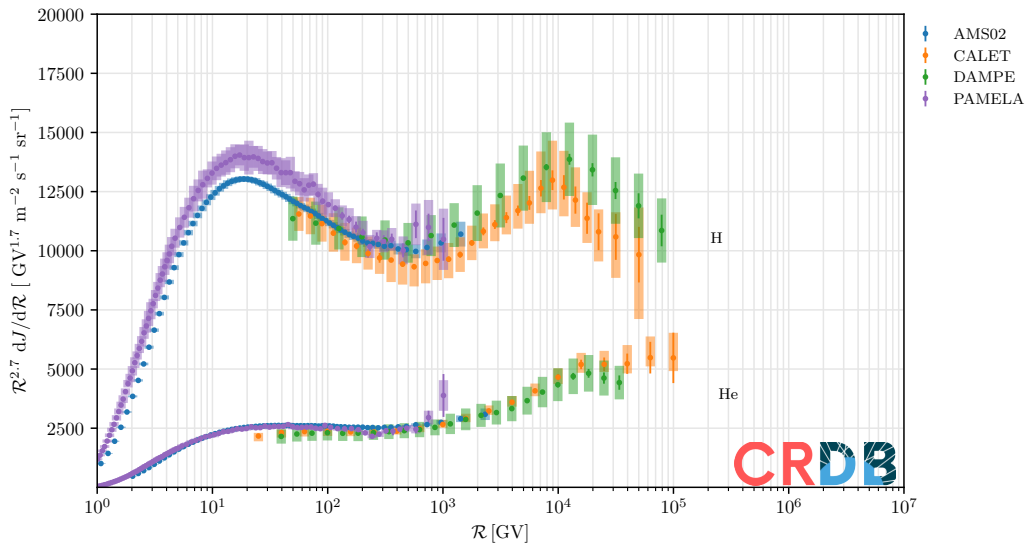
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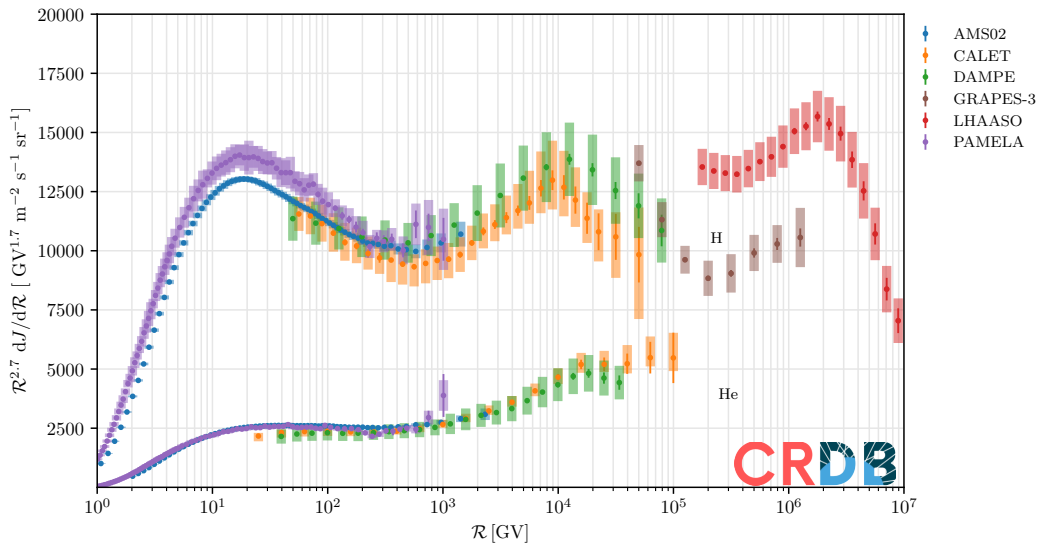
The cosmic ray spectrum

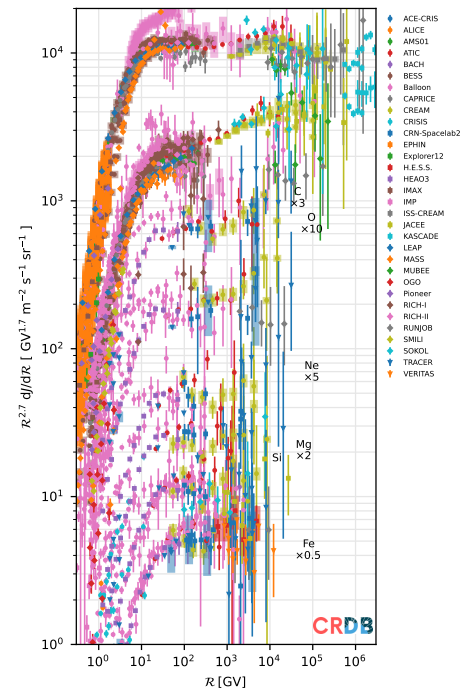


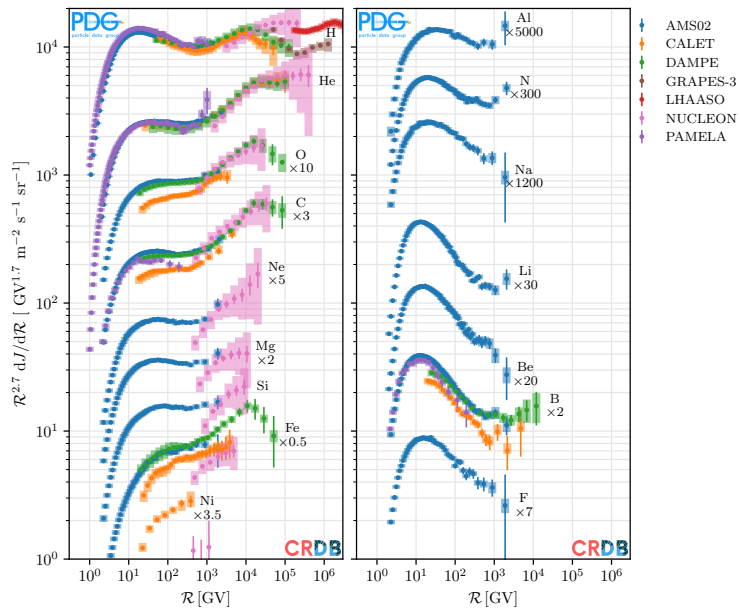
The proton and helium spectra

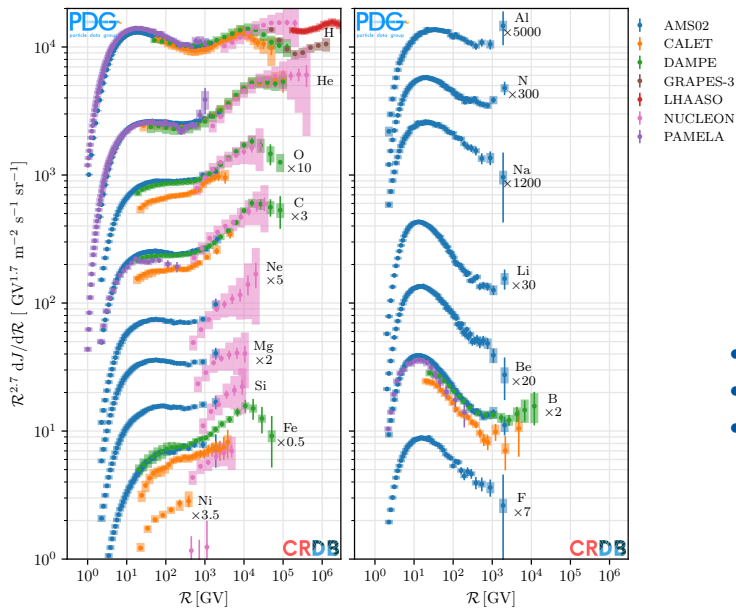


The proton and helium spectra

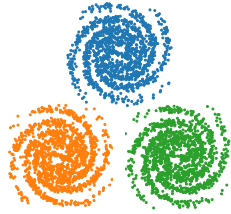




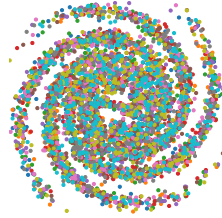




- Primaries: harder spectra
- Secondaries: softer spectra
- Same spectral breaks



Source populations



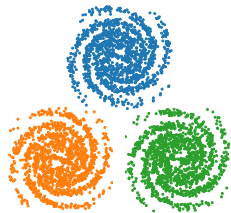
Individual sources



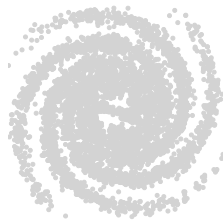
Source spectrum



Transport effects



Source populations



Individual sources

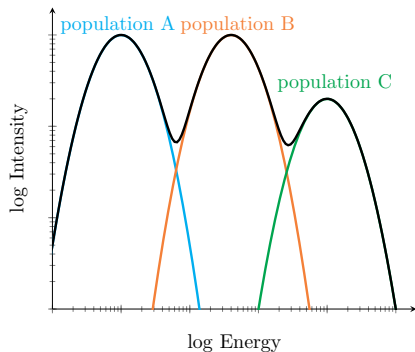


Source spectrum



Transport effects

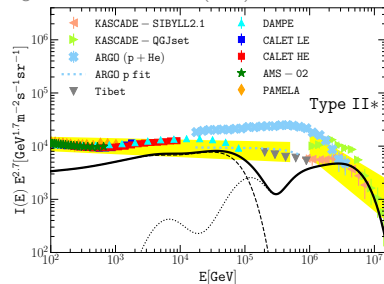
Population of sources



- Different populations of sources
 - Hard spectra, different cut-off energies
- Bumps and dips

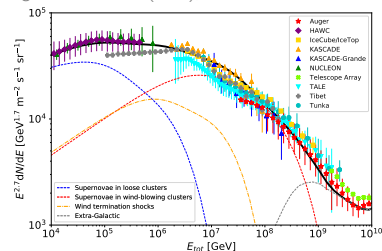
Supernova remnants

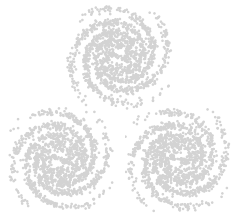
e.g. Zirakashvili & Ptuskin (2015), Cristofari, Blasi, Amato (2020)



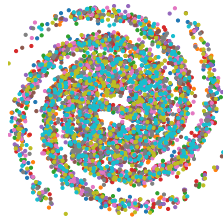
Isolated supernova remnants + star clusters

e.g. Reville & Vieu (2020),





Source populations



Individual sources



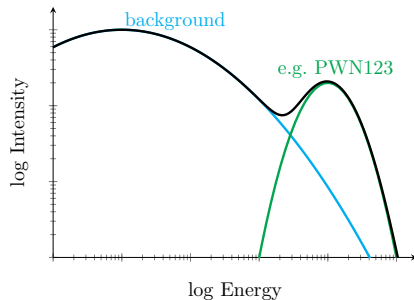
Source spectrum



Transport effects

Individual sources

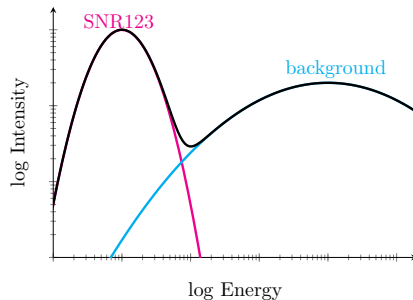
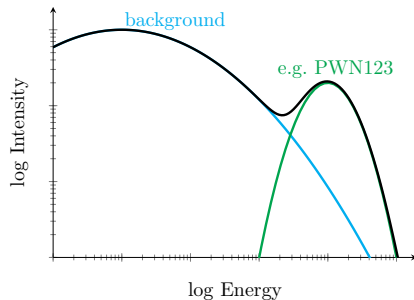
Erykin and Wolfendale (2012), Kachelrieß *et al.* (2018), Fornieri *et al.* (2020) and many more



- Background from bulk of sources
 - One (or a few) individual source(s)
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Individual sources

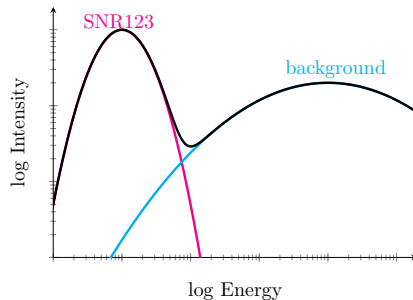
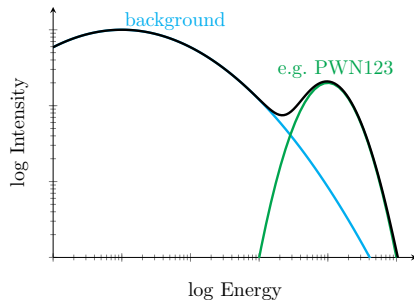
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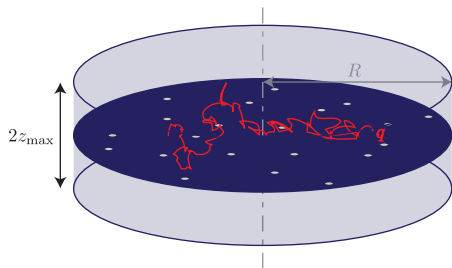


Supernova remnant paradigm

1 000 - 100 000 “active” supernova remnants in the Galaxy

Génolini *et al.* (2017)

The number of sources contributing to CR flux decreases with energy



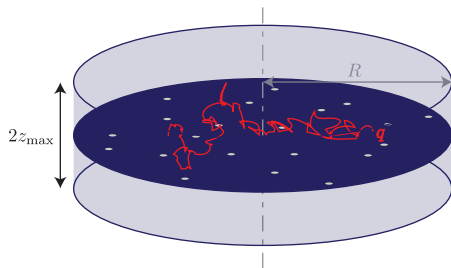
- Residence time: $t_{\text{esc}} = \frac{z_{\text{max}}^2}{2\kappa}$
- Diffusion distance: $R = \sqrt{2\kappa t_{\text{esc}}} = z_{\text{max}}$
- Source density: $\sigma = \frac{\nu t_{\text{esc}}}{\pi R_{\text{disk}}^2}$
- Source number: $N_{\text{src}} = \sigma \pi R^2 = \nu t_{\text{esc}} \frac{z_{\text{max}}^2}{R_{\text{disk}}^2}$

With typical parameters (for details → [Appendix](#)):

$$\mathcal{R} = 10 \text{ GV}, \quad 10 \text{ TV}, \quad 10 \text{ PV}$$

$$N_{\text{src}} \simeq 2 \times 10^5, \quad 2000, \quad 40$$

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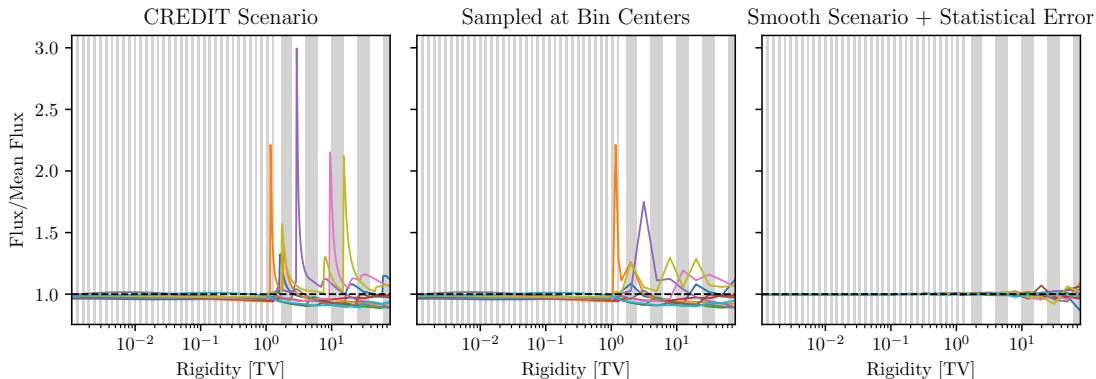
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$$N_{\text{src}} \simeq 2 \times 10^5, \quad 2000, \quad 40$$

Need to run Monte Carlo simulations
instead of cherry-picking catalogues!

Statistical errors are much smaller than CREDIT features

Stall, Loo, Mertsch (2025)

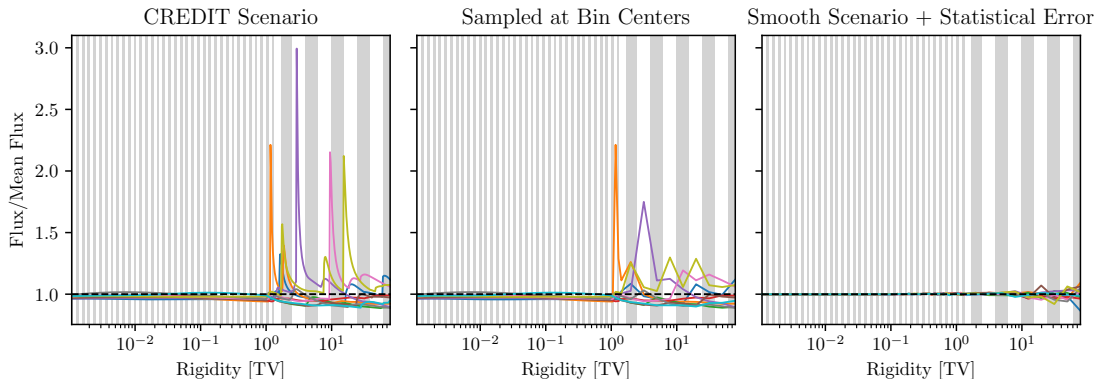
B-field amplification + **time-dependent escape** → **prominent features**



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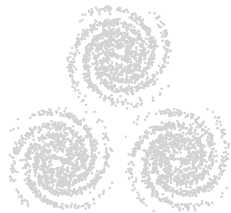
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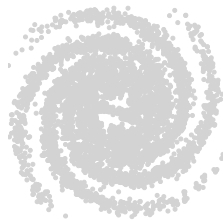


What can we learn?

- Test of field-amplification models
- Supernova paradigm



Source populations



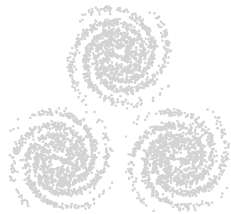
Individual sources



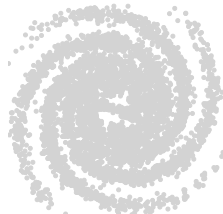
Source spectrum



Transport effects



Source populations



Individual sources



Source spectrum



Transport effects

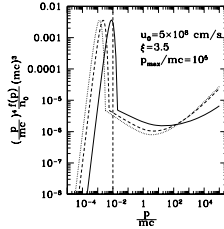
Reminder: Solution (without advection or decay)

$$\psi_j(0, \mathcal{R}) = \frac{h \left(q_j(0, \mathcal{R}) + \sum_{k>j} v n_{\text{gas}}(0) \sigma_{k \rightarrow j} \psi_k \right)}{\left(h v n_{\text{gas}}(0) \sigma_j + \frac{\kappa(\mathcal{R})}{Z_{\text{max}}} \right)}$$

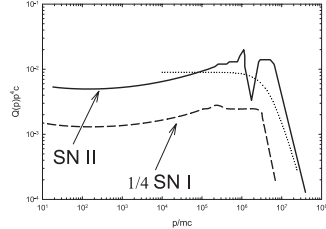


Example: concavity of non-linear diffusive shock acceleration

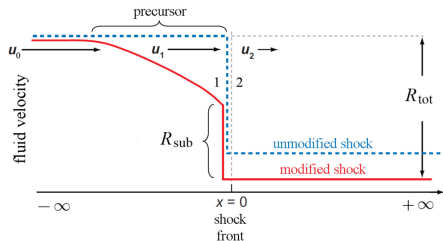
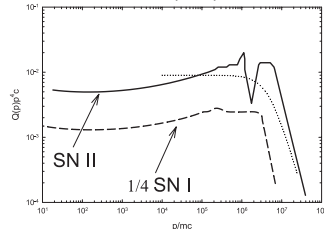
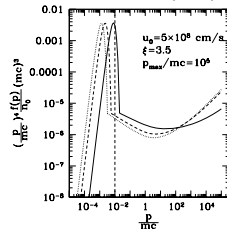
Blasi, Gabici, Vannoni (2005)



Ptuskin & Zirakashvili (2005)



Example: concavity of non-linear diffusive shock acceleration



- Spectral index γ depends on compression ratio r : $\gamma \sim \frac{3r}{r-1}$
- Low energy particles: stay close to shock $\rightarrow$ experience only subshock $\rightarrow$ soft spectrum
- High energy particles: travel further $\rightarrow$ experience full shock $\rightarrow$ hard spectrum

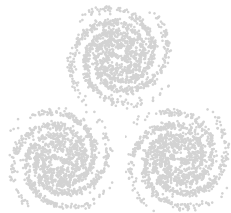
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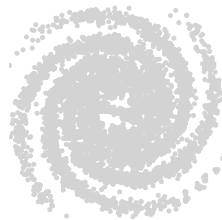


Potential issue

- $\mathcal{R}_{\text{feature}}$ depends on environmental parameters
 - Scatter in environmental parameters; time-dependencies
- Feature washed out in superposition



Source populations



Individual sources



Source spectrum



Transport effects

Reminder: Solution (without advection or decay)

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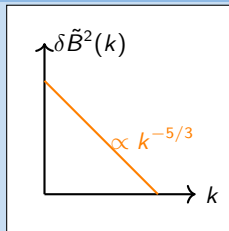
Feature in diffusion coefficient

Reminder: Solution (without advection or decay)

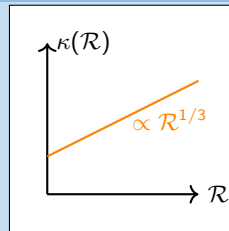
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Resonant interactions



$$\longleftrightarrow k r_g \sim 1$$



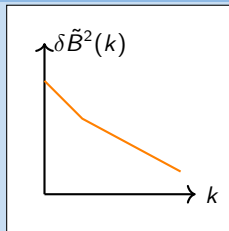
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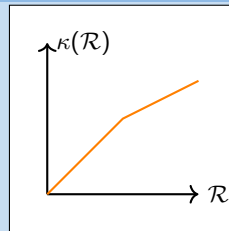
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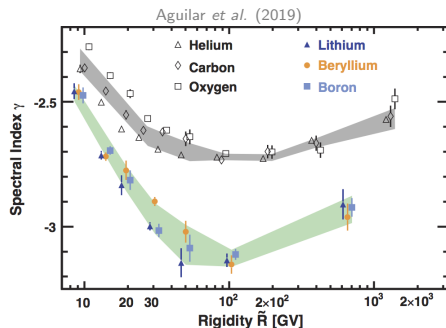
Source or transport?

- Can be distinguished by secondaries *Vladimirov et al. (2012)*

- break in source spectrum: break in secondaries similar



- break in diffusion coefficient: break in secondaries $\sim 2\times$ as strong



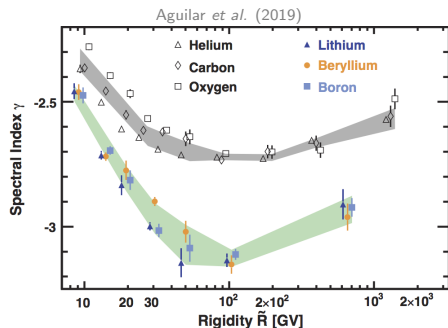
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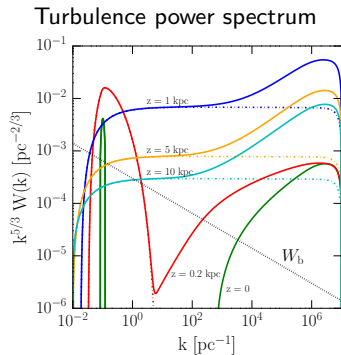


What can we learn?

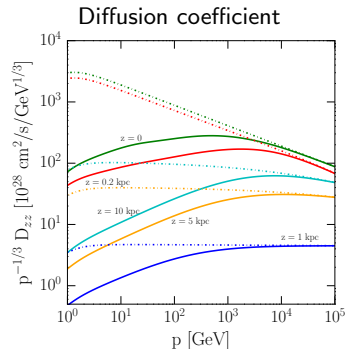
The break is in the diffusion coefficient!

Streaming instability and large-scale transport

Evoli *et al.*, PRD **99** (2019) 103023

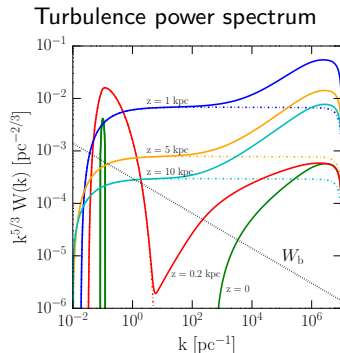


$$\left(\frac{k}{10^6 \text{ pc}^{-1}} \right) \sim \left(\frac{p}{\text{GeV}} \right)^{-1}$$

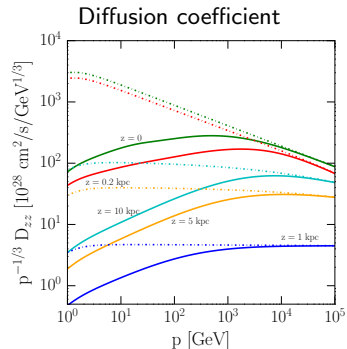


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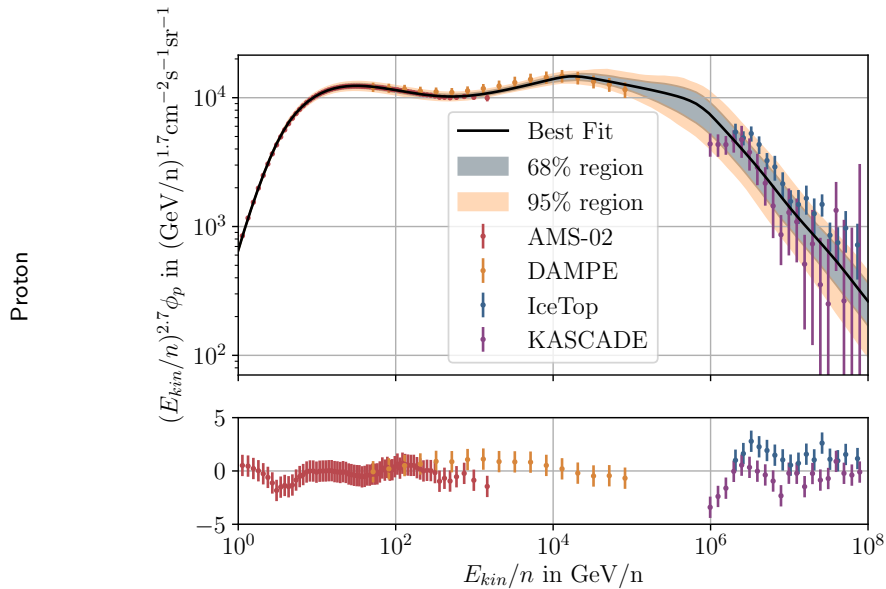


What can we learn?

- The power spectrum of magnetised turbulence has a break at $k_{\text{br}} \simeq 3000 \text{ pc}^{-1}$
- Turbulence generated by $\begin{cases} \text{supernovae} & \text{for } k \lesssim k_{\text{br}} \\ \text{cosmic rays} & \text{for } k \gtrsim k_{\text{br}} \end{cases}$
- Important consequences for galaxy formation and evolution

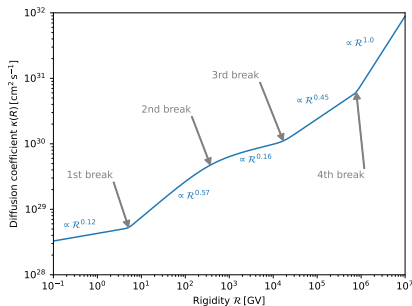
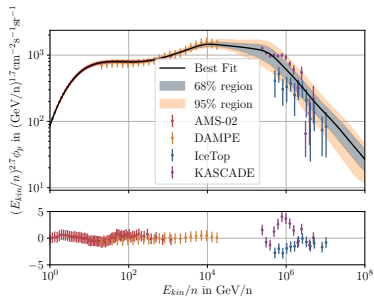
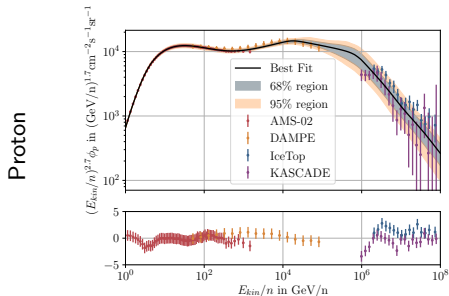
Anatomy of a diffusion coefficient

Schwefer, Mertsch, Wiebusch (2022)



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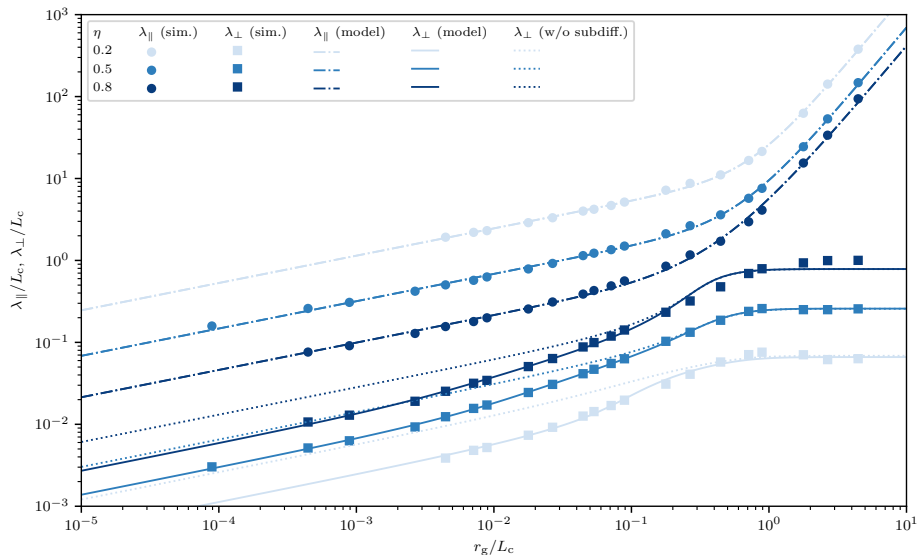
Schwefer, Mertsch, Wiebusch (2022)



- 1. Break: Hardening of spectra at low energies due to stochasticity effect *Phan et al. (2021)* (for details → [Appendix](#)):
- 2. Break: Transition from self-generated to external turbulence *Blasi, Amato, Serpico (2012)*
- 3. Break: ?!
- 4. Break: Transition from resonant to small-angle scattering

Mean-free paths

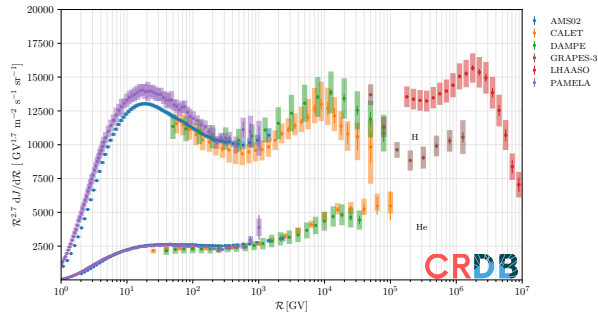
Kuhlen, Mertsch, Phan (2025)



Outline

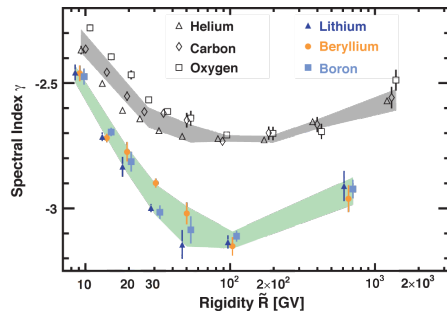
- ① Introduction
- ② Fundamental observations and key insights
- ③ Standard picture of cosmic rays
- ④ Measurements and interpretation 1: Spectral breaks
- ⑤ Measurements and interpretation 2: Non-universality
- ⑥ Measurements and interpretation 3: Spectral coincidences
- ⑦ Measurements and interpretation 4: Electrons and positrons
- ⑧ Summary

Non-universality



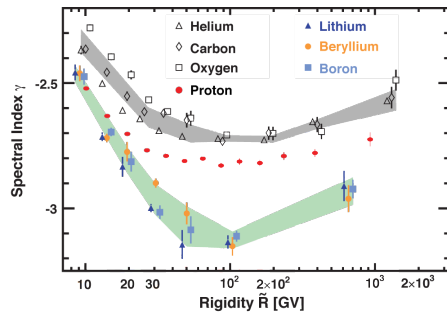
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- Spectral indices shifted by ~ 0.1 😞

Non-universality



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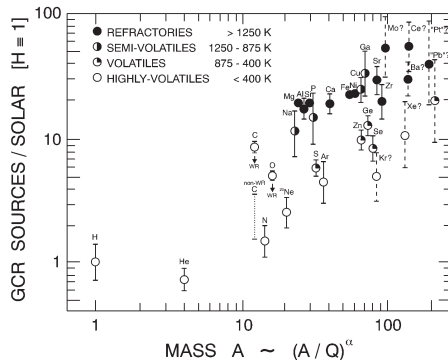
Non-universality



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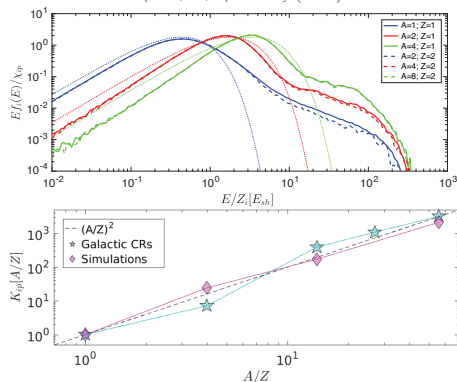
Species-dependence in DSA?

Meyer, Drury, Ellison (1997)



Source abundances $\sim$ solar, but with mass-dependent trend

Caprioli, Yi, Spitkovsky (2017)



Injection into DSA depends on A/Z , but spectral shapes are the same

Environmental effect

Ohira & Ioka (2010)

1 $p_{\max}$ decreases with time or R_{sh} :

$$p_{\max}(R_{\text{sh}}) \propto R_{\text{sh}}^{-\alpha}, \quad \text{e.g.} \quad p_{\max} = Zp_{\text{knee}} \left(\frac{R_{\text{sh}}}{R_{\text{Sedov}}} \right)^{-\alpha}; \quad \text{e.g.} \quad \alpha = 6.5$$

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Inverse function: $p_{\max}^{-1}(p) \propto p^{-1/\alpha}$

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$$F_{\text{SNR}}(R_{\text{sh}}, p) \propto p^{-s} R_{\text{sh}}^{\beta}; \quad \text{e.g.} \quad \beta = 1.5$$

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$$\Rightarrow F_{\text{esc}}(p) = F_{\text{SNR}}(p_{\max}^{-1}(p), p) \propto p^{-s} \left(p^{-1/\alpha} \right)^{\beta} = p^{-(s + \frac{\beta}{\alpha})}$$

- 1 $p_{\max}$ decreases with time or R_{sh} :

$$p_{\max}(R_{\text{sh}}) \propto R_{\text{sh}}^{-\alpha}, \quad \text{e.g.} \quad p_{\max} = Zp_{\text{knee}} \left(\frac{R_{\text{sh}}}{R_{\text{Sedov}}} \right)^{-\alpha}; \quad \text{e.g.} \quad \alpha = 6.5$$

Inverse function: $p_{\max}^{-1}(p) \propto p^{-1/\alpha}$

- 2 Spectrum inside SNR: a power-law but normalisation increases with R_{sh}

$$F_{\text{SNR}}(R_{\text{sh}}, p) \propto p^{-s} R_{\text{sh}}^{\beta}; \quad \text{e.g.} \quad \beta = 1.5$$

- 3 CRs escaping in $[R_{\text{sh}}, R_{\text{sh}} + dR_{\text{sh}}] =$ Escaped CRs in $[p_{\max}, p_{\max} + dp]$:

$$F_{\text{SNR}}(R_{\text{sh}}, p_{\max}) \left| \frac{dp_{\max}}{dR_{\text{sh}}} \right| dR_{\text{sh}} = F_{\text{esc}}(p_{\max}) dp_{\max}$$

$$\Rightarrow F_{\text{esc}}(p) = F_{\text{SNR}}(p_{\max}^{-1}(p), p) \propto p^{-s} \left(p^{-1/\alpha} \right)^{\beta} = p^{-(s + \frac{\beta}{\alpha})}$$

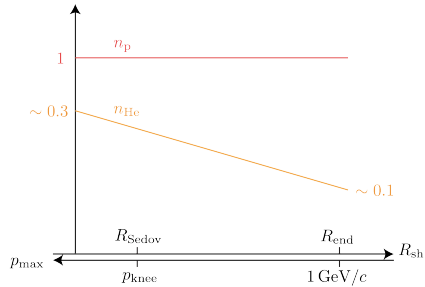
- 4 s, α depend on environment, β can depend on species

Environmental effect

Ohira & Ioka (2010)

$$\Rightarrow F_{\text{esc}}(p) = F_{\text{SNR}}(p_{\text{max}}^{-1}(p), p) \propto p^{-s} \left(p^{1/\alpha} \right)^{-\beta} = p^{-\left(s + \frac{\beta}{\alpha}\right)}$$

Abundance relative to p

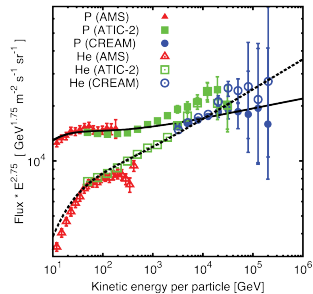
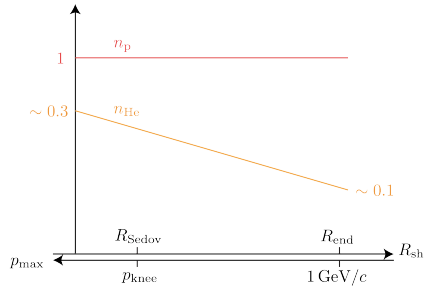


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Abundance relative to p



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Exotic scenarios

Lipari (2014, 2019); Blum *et al.* (2009, 2019)

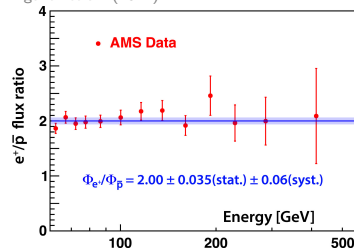
- $\bar{p}$ propagate **without** energy losses above few GeV
- $e^\pm$ suffer **strong** radiative losses above few GeV
- e^+ spectrum should be softer than $\bar{p}$ spectrum

- Yet, spectra shapes are markedly similar above few tens of GeV
- Also,

$$\frac{\phi_{e^+}}{\phi_{\bar{p}}} \approx \frac{\sigma_{p \rightarrow e^+}}{\sigma_{p \rightarrow \bar{p}}}$$

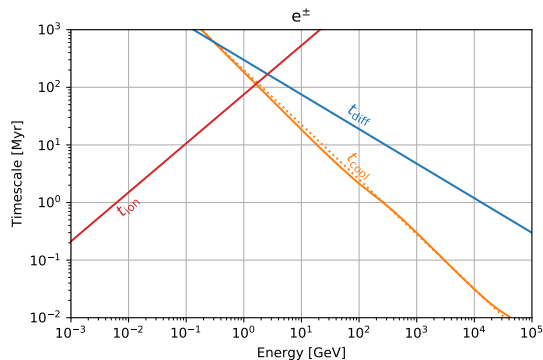
→ Maybe energy losses are actually small for $e^\pm$ below $\sim \text{TeV}$?

Aguilar *et al.* (2021)



Reduced residence time

Time scales:



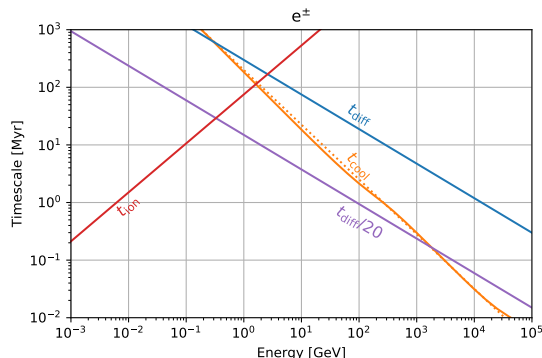
- $t_{\text{diff}} = \frac{z_{\text{max}}^2}{2\kappa}$ with $z_{\text{max}} = 5$ kpc, $\kappa(10 \text{ GV}) = 5 \times 10^{28} \text{ cm}^2 \text{ s}^{-1}$
- t_{cool} : KN cross-section with $\rho = \{0.26, 0.6, 0.6, 0.1\} \text{ eV cm}^{-3}$ for CMB, IR, opt, UV; $3 \mu\text{G}$ B-field
- t_{ion} : $n_{\text{H}} = 0.5 \text{ cm}^{-3}$ (WIM) and $n_{\text{H}} = 0.5 \text{ cm}^{-3}$ (WNM) and 100 pc wide gas disk

In a diffusion model with $E^{-\Gamma}$ sources in disk:

- $\phi(E) \propto E^{-\Gamma-\delta}$ if diffusion dominated
- $\phi(E) \propto E^{-\Gamma-(\delta+1)/2}$ if cooling dominated

Reduced residence time

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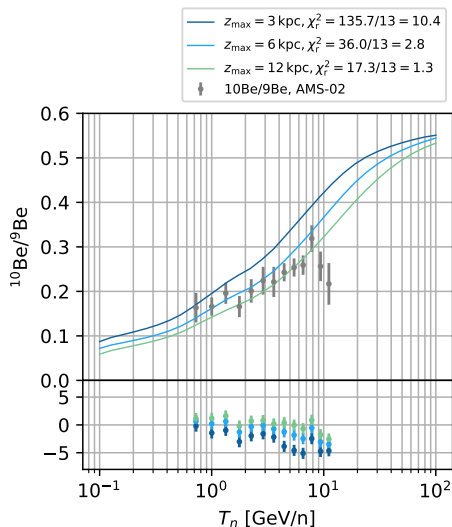


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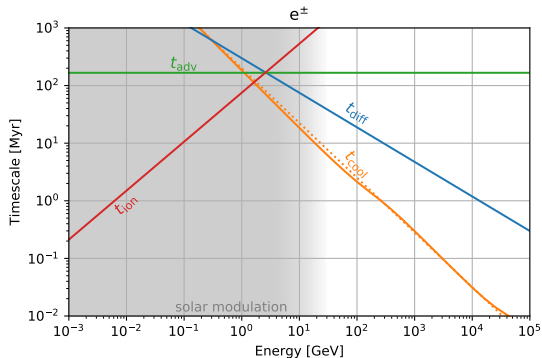
Cosmic ray clocks



- Fit to p, He, C, O, B/C with fixed $z_{\max}$
- Pure diffusion model:
 - diffusion coefficient with 2 breaks
 - no advection
 - no reacceleration
- Pure power law sources spectra
- Solar modulation: force-field approach

Another argument: cooling break

- Transition escape $\rightarrow$ cooling leads to spectral break
- If $t_{\text{diff}}(1\text{GeV}) \sim t_{\text{cool}}(1\text{GeV}) \rightarrow$ hidden due to other transport processes
- If $t_{\text{diff}}(1\text{TeV}) \sim t_{\text{cool}}(1\text{TeV}) \rightarrow$ could be visible

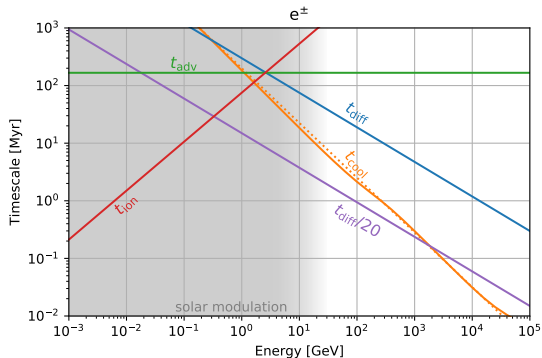


Could also be a stochasticity effect: extended transition around E_1 where

$$\ell_{\text{loss}} \simeq 2\sqrt{\kappa(E_1)t_{\text{cool}}(E_1)} \sim \text{typical source distance}$$

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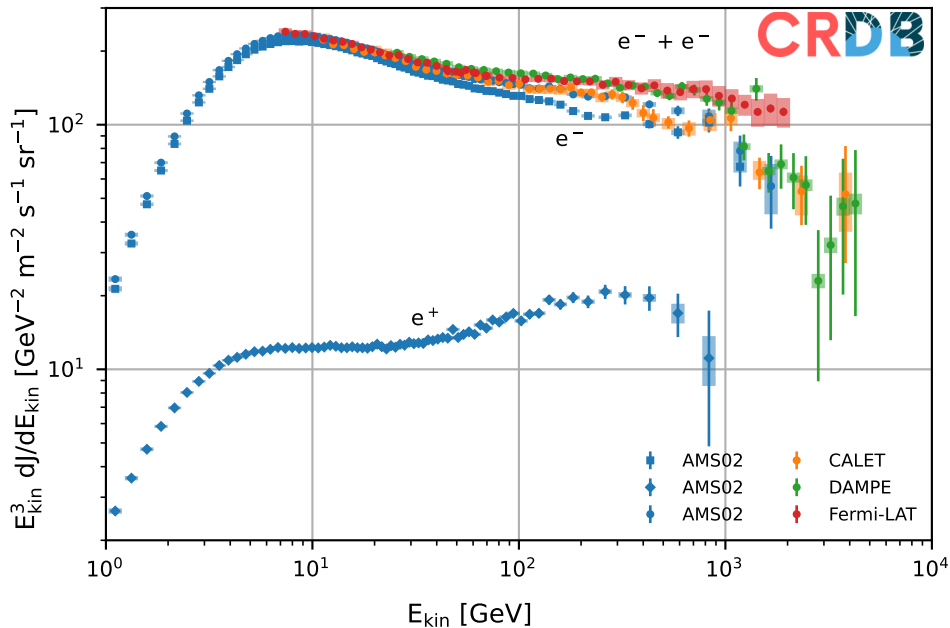
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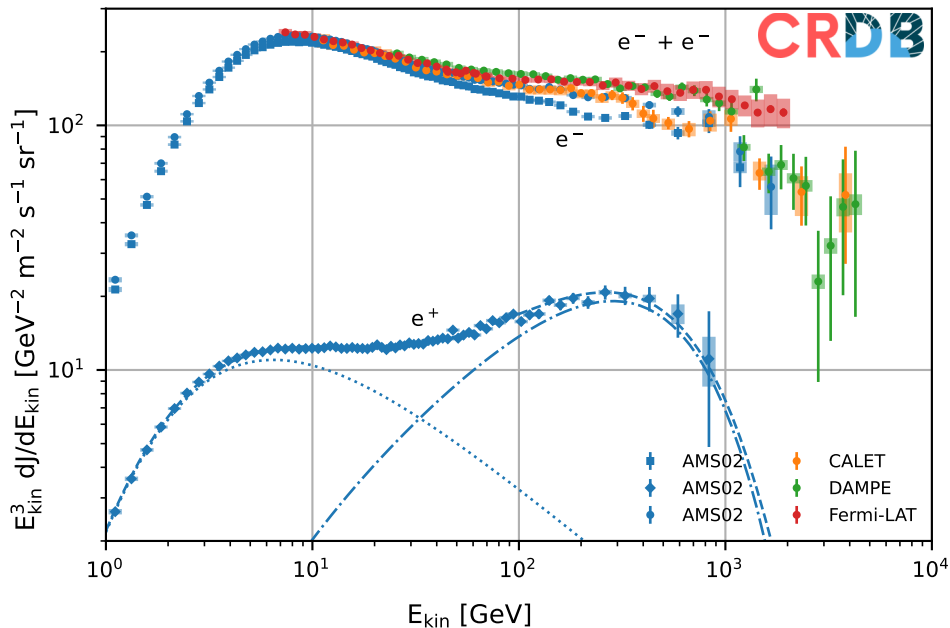
$e^+ + e^-$ data

Ambrosi *et al.* (2017); Abdollahi *et al.* (2017); Aguilar *et al.* (2021); Torri *et al.* (2021)



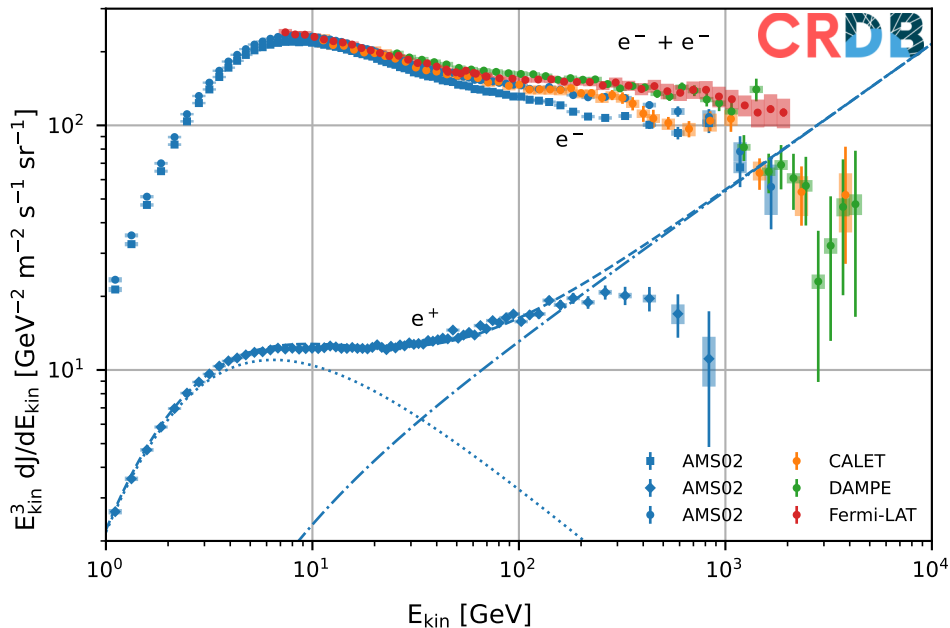
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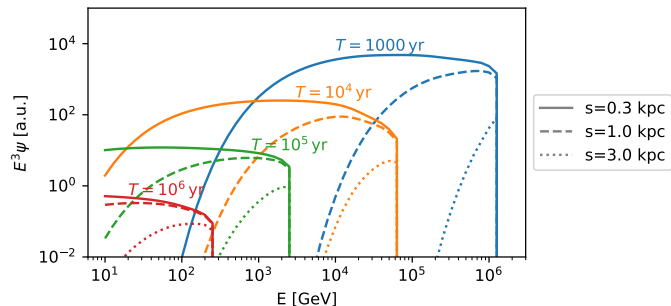
Cut-off in positrons

- Cut-off is sensitive to source spectrum, e.g. $Q(E_0) \propto E_0^{-\Gamma}$

What can we learn?

The acceleration of positrons is limited to ~ 10 TeV.

→ Info on age, environment, radiation fields



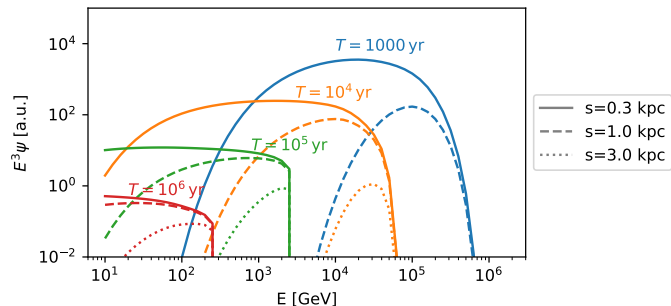
Cut-off in positrons

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The acceleration of positrons is limited to $\sim 10 \text{ TeV}$.

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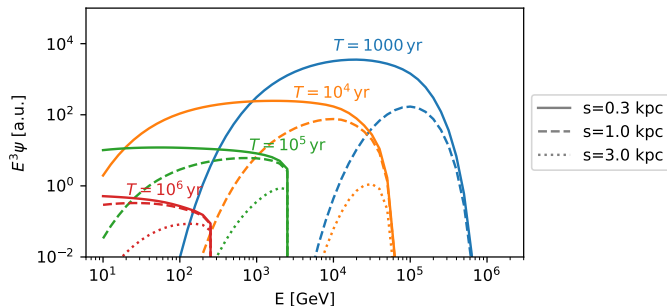
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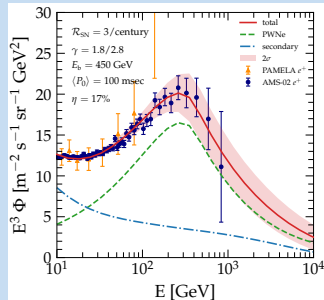
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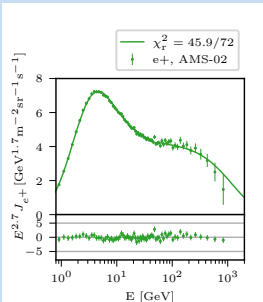
Pulsars

e.g. Evoli et al. (2020)



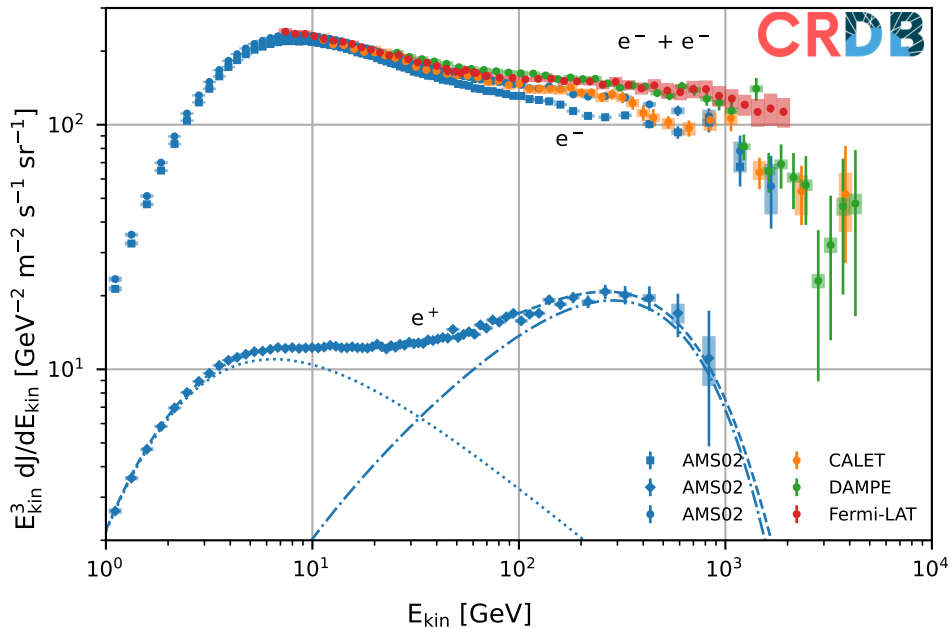
Other astro sources

e.g. Mertsch, Sarkar, Vittino (2020)



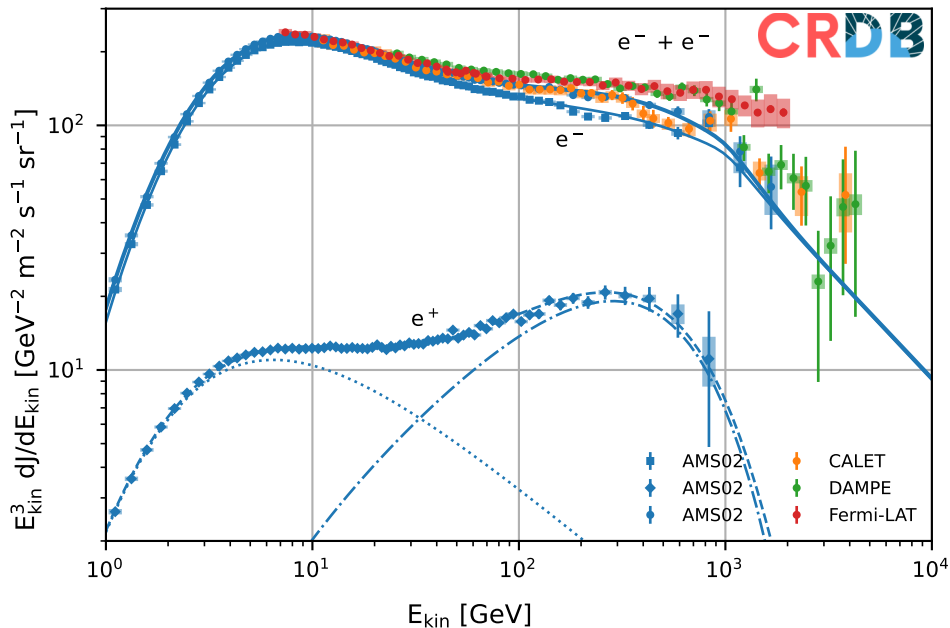
$e^+ + e^-$ data

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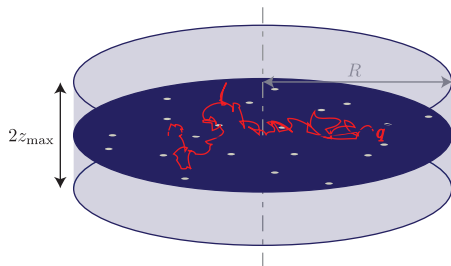


$e^+ + e^-$ data

Ambrosi *et al.* (2017); Abdollahi *et al.* (2017); Aguilar *et al.* (2021); Torri *et al.* (2021)



The number of sources contributing to CR flux decreases with sharply energy



- Residence time: $t_{\text{cool}} = \frac{E}{\dot{E}}$
- Diffusion distance: $R = \sqrt{2\kappa t_{\text{cool}}}$
- Source density: $\sigma = \frac{\nu t_{\text{cool}}}{\pi R_{\text{disk}}^2}$
- Source number: $N_{\text{src}} = \sigma \pi R^2 = \nu t_{\text{cool}} \frac{2\kappa t_{\text{cool}}}{R_{\text{disk}}^2}$

With typical parameters (for details → [Appendix](#)):

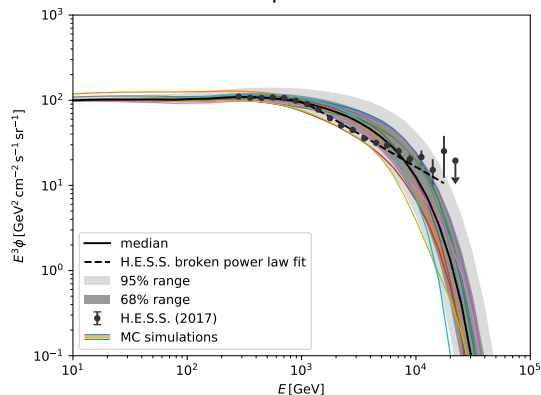
$$\mathcal{R} = 10 \text{ GV}, \quad 10 \text{ TV}, \quad 10 \text{ PV}$$

$$N_{\text{src}} \simeq 2 \times 10^4, \quad 1, \quad 10^{-4}$$

Spectral shape and source rate

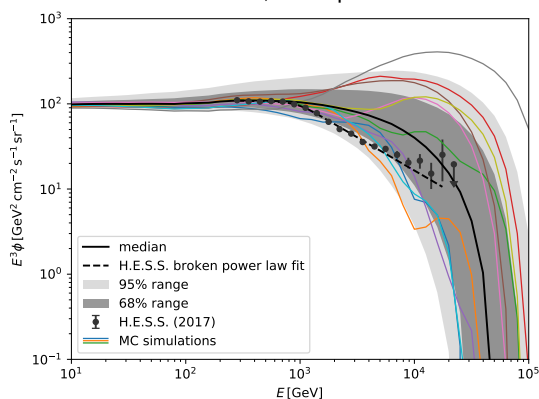
Mertsch (2018)

Canonical supernova rate



Statistically disfavoured

Reduced rate $\rightarrow$ fewer, more powerful sources

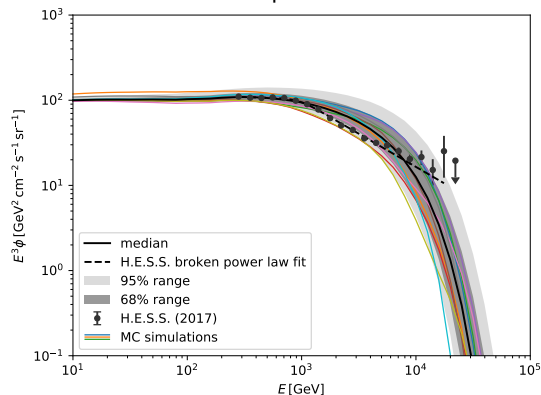


Statistically preferred

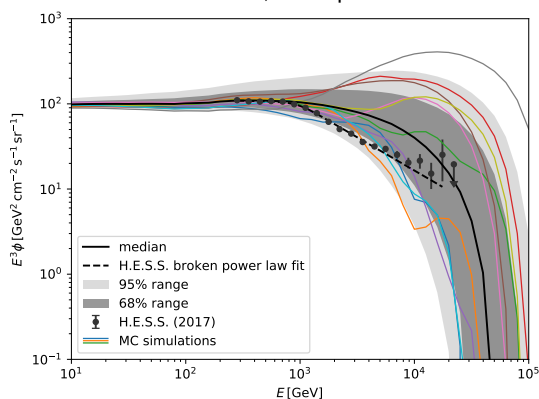
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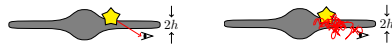
Statistically preferred

What can we learn?

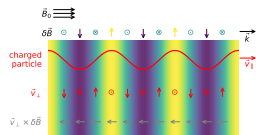
The TeV break indicates lower source rate, e.g. super bubbles

Outline

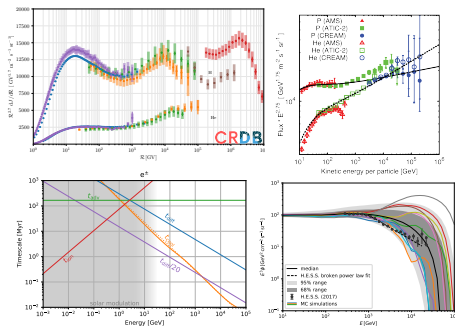
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Insights into Galactic CRs from spectrum, abundances, anisotropies



Diffusion $\rightarrow$ basic understanding of wave-particle interactions



Anomalies:

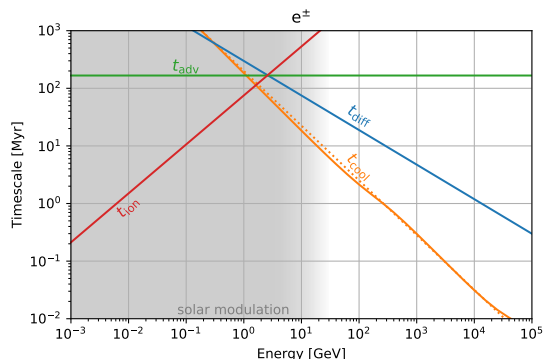
- 1 Spectral breaks
- 2 Non-universality
- 3 Spectral coincidences
- 4 High-energy electrons

Outline

9 Backup

Time scales

Time scales:



- $t_{\text{diff}} = \frac{z_{\text{max}}^2}{2\kappa}$ with $z_{\text{max}} = 5 \text{ kpc}$, $\kappa(10 \text{ GV}) = 5 \times 10^{28} \text{ cm}^2 \text{ s}^{-1}$
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Source density is oftentimes considered smooth, while it really is discrete

Transport equation

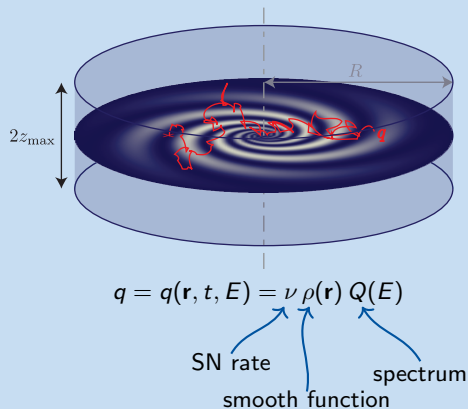
$$\frac{\partial \psi}{\partial t} - \nabla \cdot \kappa \cdot \nabla \psi + \dots = q$$

Source density is oftentimes considered smooth, while it really is discrete

Transport equation

$$\frac{\partial \psi}{\partial t} - \nabla \cdot \kappa \cdot \nabla \psi + \dots = q$$

Smooth source density

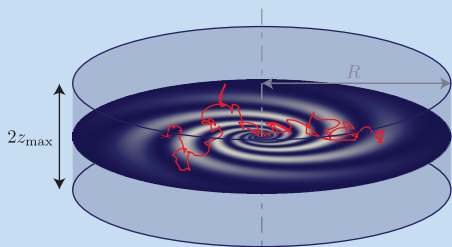


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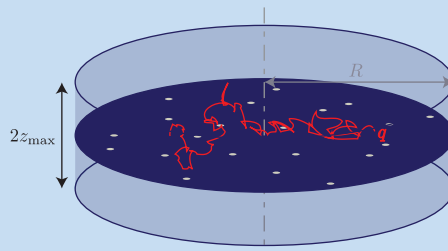
$$q = q(\mathbf{r}, t, E) = \nu \rho(\mathbf{r}) Q(E)$$

SN rate

smooth function

spectrum

Discrete sources



$$q = q(\mathbf{r}, t, E) = \sum_i \delta^{(3)}(\mathbf{r} - \mathbf{r}_i) \delta(t - t_i) Q(E)$$

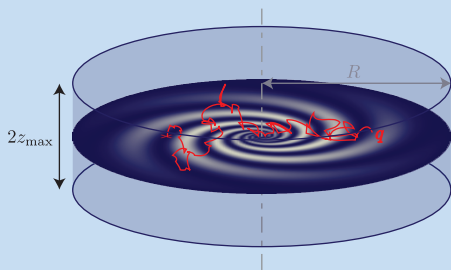
$\mathbf{r}_i$ drawn from $\rho(\mathbf{r})$ and t_i from uniform distribution

Source density is oftentimes considered smooth, while it really is discrete

Transport equation

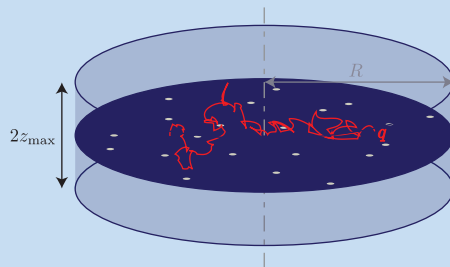
$$\frac{\partial \mathbf{G}}{\partial t} - \nabla \cdot \kappa \cdot \nabla \mathbf{G} + \dots = \delta^{(3)}(\mathbf{r} - \mathbf{r}_i) \delta(t - t_i) Q(E)$$

Smooth source density



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Discrete sources



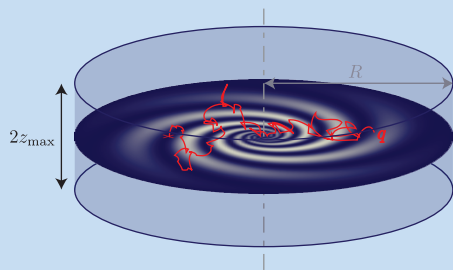
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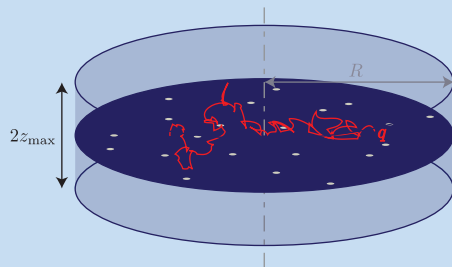
Smooth source density



$$q = q(\mathbf{r}, t, E) = \nu \rho(\mathbf{r}) Q(E)$$

$$\psi(\mathbf{r}, t, E) = \int d^3\mathbf{r}' dt' \nu \rho(\mathbf{r}') G(\mathbf{r} - \mathbf{r}', t - t', E)$$

Discrete sources



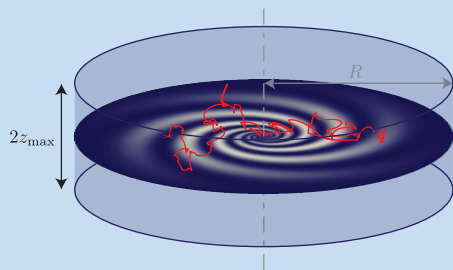
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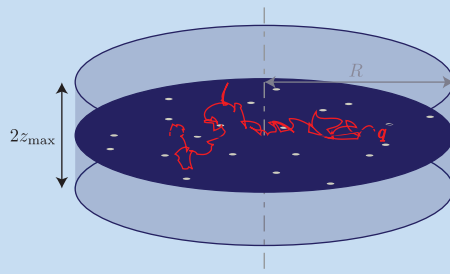
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Stochastic nature of sources implies fluctuations in spectrum

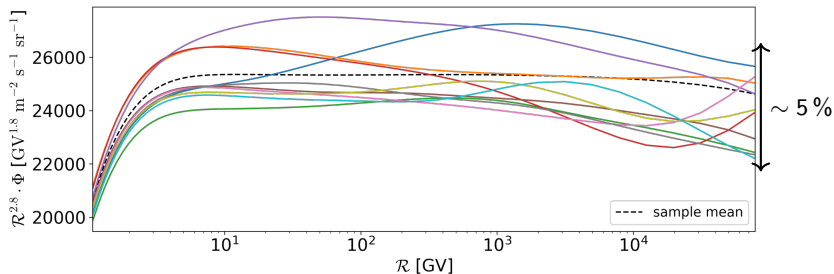
- Solution: $\psi(\mathbf{r}, t, p) = \sum_i G(\mathbf{r} - \mathbf{r}_i, t - t_i, E)$
- $\mathbf{r}_i, t_i$ are random variables $\Rightarrow \psi(\mathbf{r}, t, p)$ is random variable
- Mean: $\langle \psi(\mathbf{x}, t, p) \rangle = \int d^3\mathbf{r}' dt' \nu \rho(\mathbf{r}') G(\mathbf{r} - \mathbf{r}', t - t', E)$

Can we use $\psi - \langle \psi \rangle$ to find sources?

Stochastic nature of sources implies fluctuations in spectrum

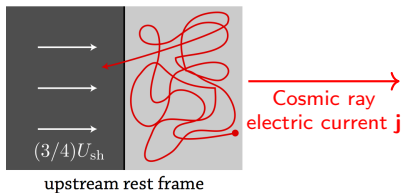
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The Bell instability can amplify B-fields

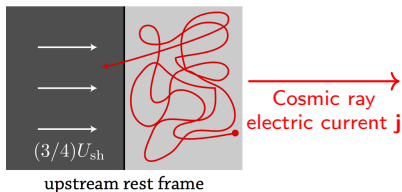
Bell (2004)



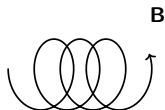
- If B -field too weak, particles escape
- Electric current $\mathbf{j}$
- Waves modes unstable in the presence of current $\mathbf{j}$

The Bell instability can amplify B-fields

Bell (2004)



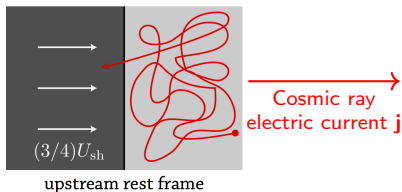
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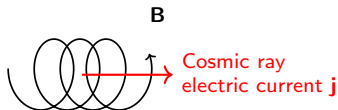
- CRs with gyroradius r_g tied to field lines
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- $\mathbf{B}$ -field density satisfies $\varepsilon_B \sim \frac{U_{sh}}{c} \varepsilon_{CR}$

The Bell instability can amplify B-fields

Bell (2004)



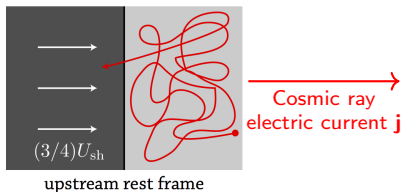
- If B -field too weak, particles escape
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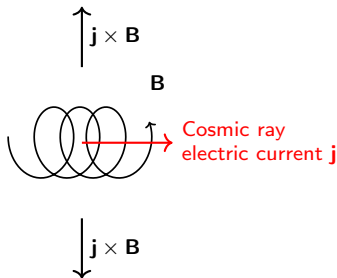
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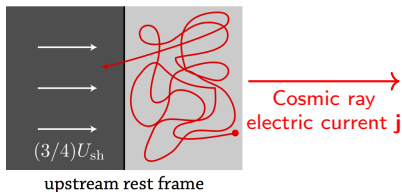
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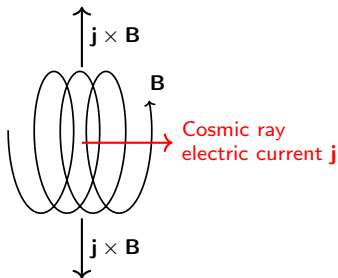
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Bell instability makes optimal use of $\mathbf{j} \times \mathbf{B}$ force

Slide concept: T. Bell

- Growth rate:

$$\gamma \sim \sqrt{\frac{jBk}{\rho}} \quad \text{or} \quad \frac{\gamma^2}{k} \sim \frac{jB}{\rho}$$

Wave number



- Compare to acceleration of fluid element of size $z \sim 1/k$ in time $t \sim 1/\gamma$:

$$\frac{z}{t^2} \sim \frac{1}{\rho} |\mathbf{j} \times \mathbf{B}| \lesssim \frac{jB}{\rho} \quad \rightarrow \quad \frac{\gamma^2}{k} \lesssim \frac{jB}{\rho}$$

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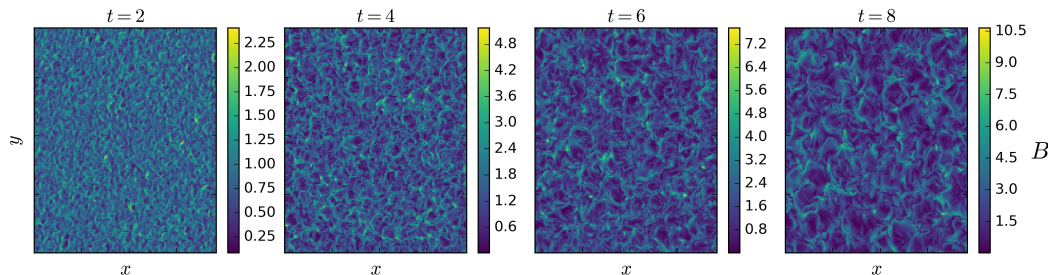
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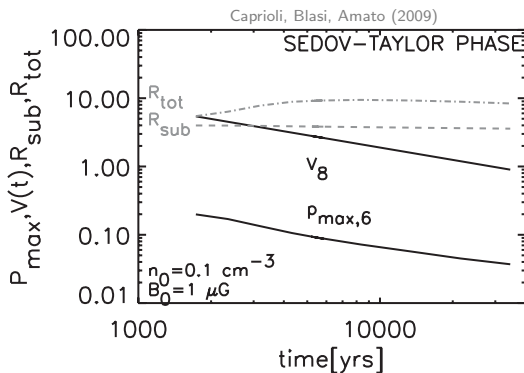
Matthews *et al.* (2017)



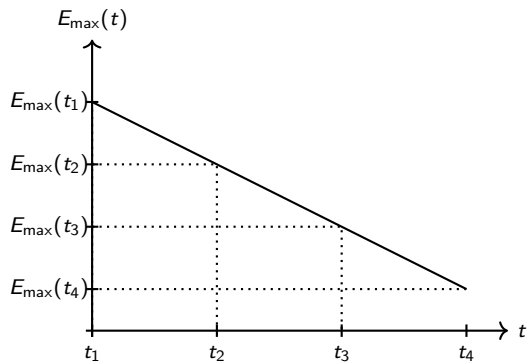
Grows rapidly on small scales

The highest CR energies can be achieved at start of Sedov-Taylor phase

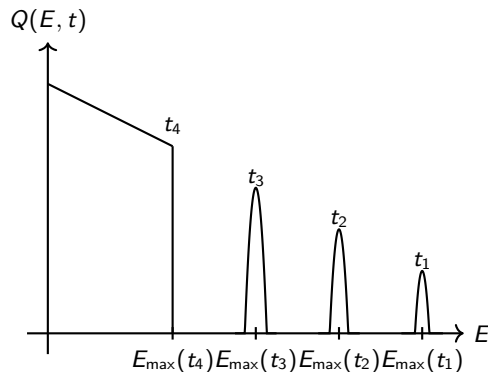
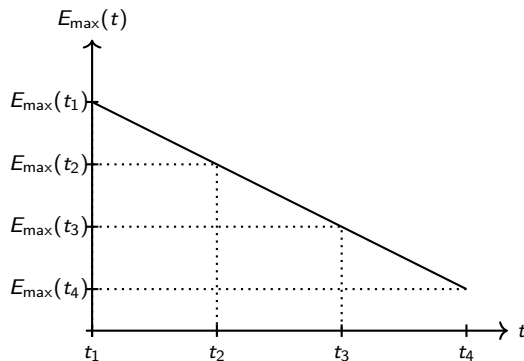
- Shock speed U_{sh} enters into growth rate γ through escape current j
 - Saturation field $B \propto U_{\text{sh}}^{3/2}$
 - $U_{\text{sh}} \propto t_{\text{age}}^{-3/5}$
- $E_{\text{max}} \propto U_{\text{sh}}^2 B t_{\text{age}} \propto t_{\text{age}}^{-11/10}$



Time-dependence of B-field amplification determines CR escape



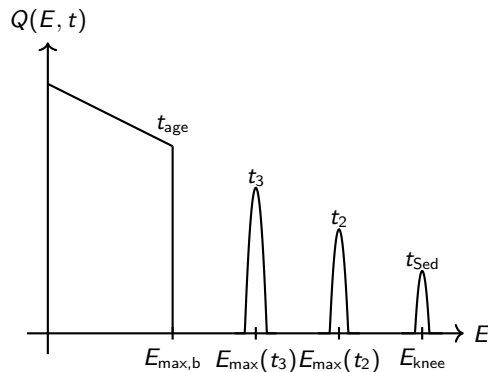
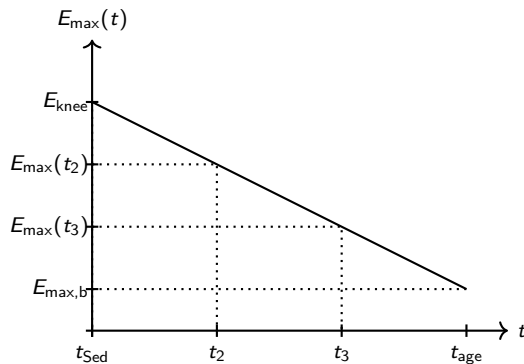
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also Gabici, Aharonian, Casanova (2009); Caprioli, Blasi, Amato (2010);
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- $E_{\max}$ decreases with time
- At any one time t , particles of energy $E_{\max}(t)$ escape
- Ultimately, all particles with $E < E_{\max,b}$ escape

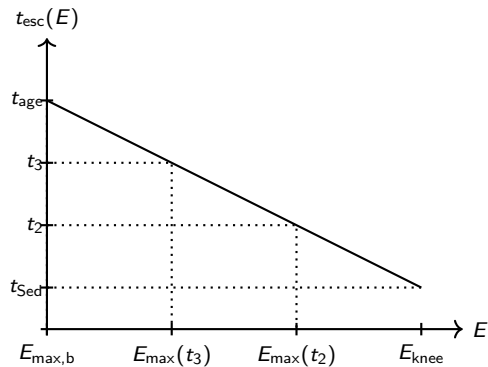
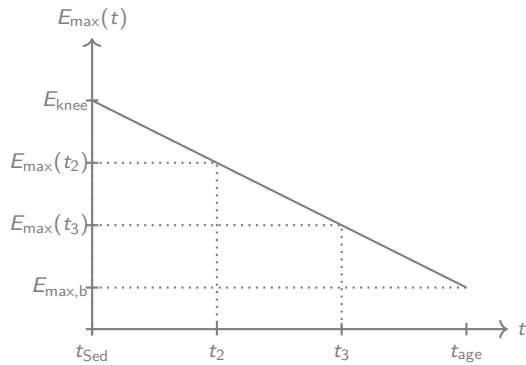
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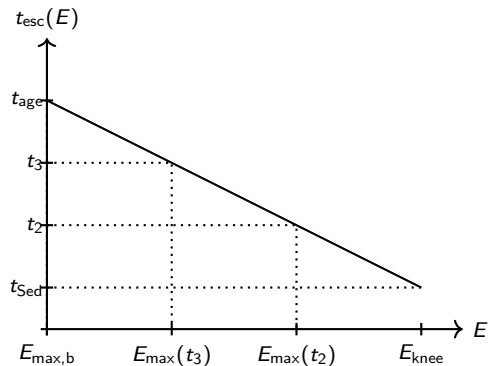
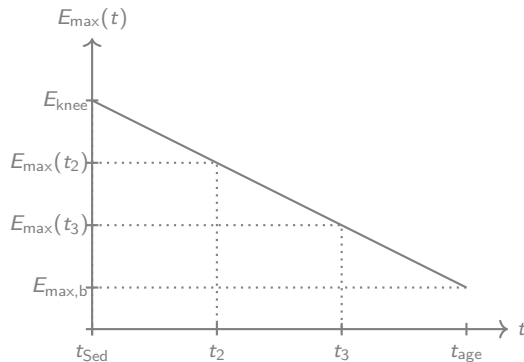
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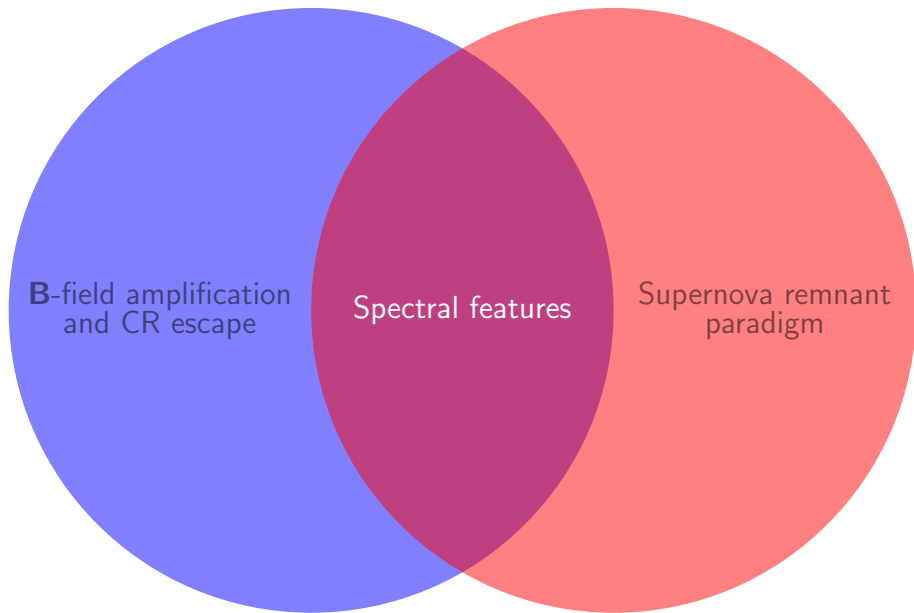
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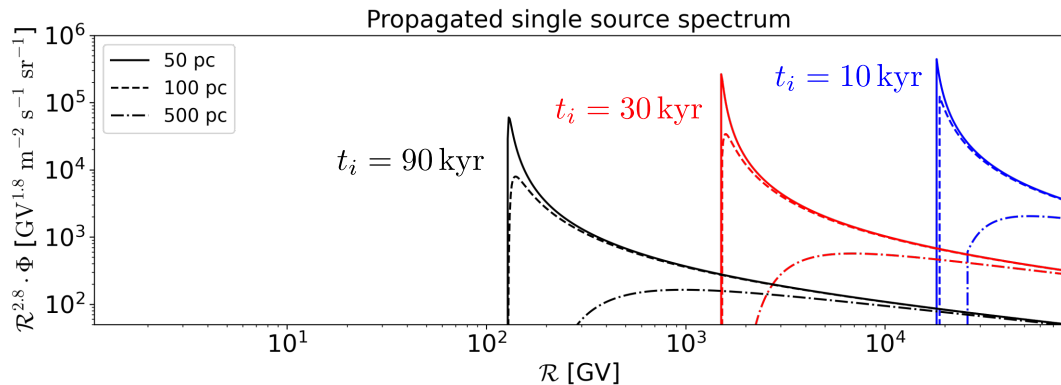


**Cosmic-Ray Energy-Dependent Injection Time
(CREDIT) scenario**



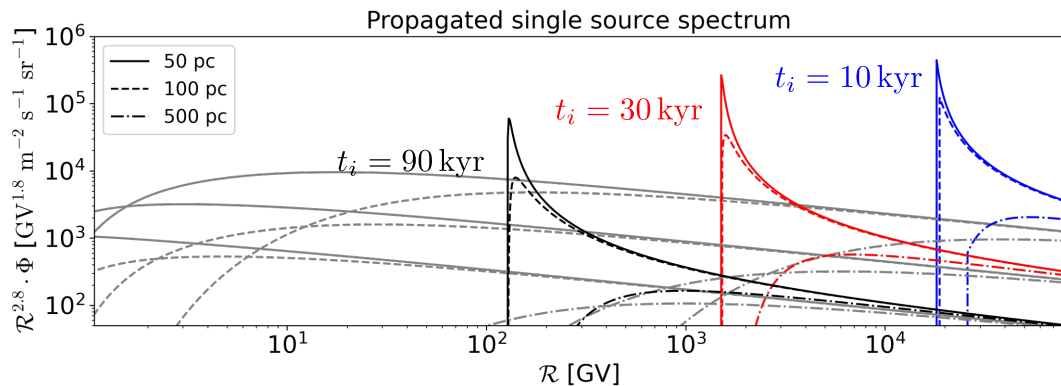
The Green's function has narrow spectral features

$$\frac{\partial G}{\partial t} - \nabla \cdot \kappa \cdot \nabla G = \delta^{(3)}(\mathbf{r} - \mathbf{r}_i) \delta(t - t_i - t_{\text{esc}}(E)) Q(E)$$



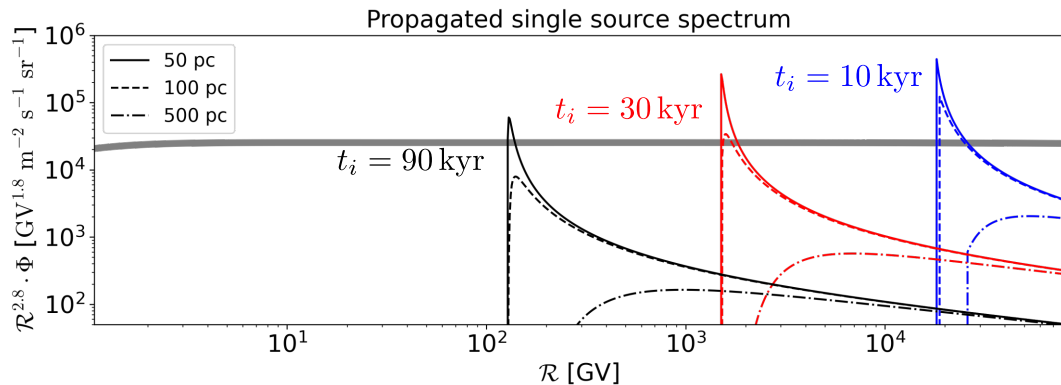
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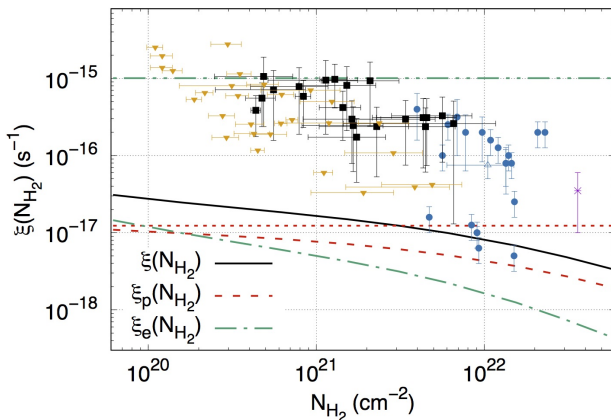


The ionisation puzzle



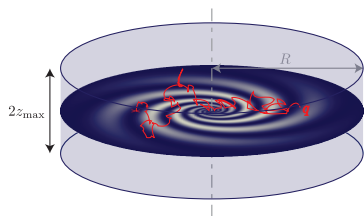
- Cosmic rays are ionising dense molecular clouds
- But the observed ionisation is much higher than expected
- Maybe the flux of cosmic rays elsewhere is different from what it is here?!

Phan, Morlino, Gabici (2018)

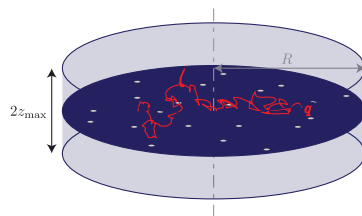


Source discreteness

Phan, Schulze, Mertsch, Recchia, Gabici (2021)



$$\psi(\mathbf{x}, t, p) = \int d\mathbf{x}' dt' \mathcal{G}(\mathbf{x} - \mathbf{x}', t - t') \underbrace{s(\mathbf{x}', t')}_{\text{src. density}} s_p(p)$$



$$\psi(\mathbf{x}, t, p) = \sum_n \mathcal{G}(\mathbf{x} - \mathbf{x}_n, t - t_n) s_p(p)$$

- Assuming smooth source density is good approximation if
(diffusion-loss length) $\gg$ (average source separation)
- However, energy losses can severely limit diffusion-loss length
- Examples:
 - High-energy $e^\pm$
 - Low-energy CRs

A statistical model

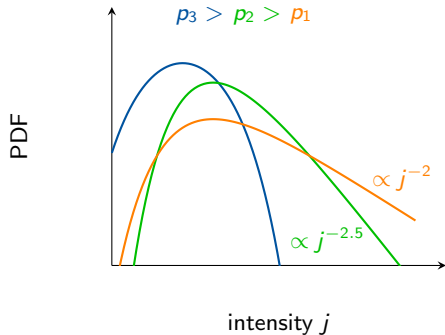
Phan, Schulze, Mertsch, Recchia, Gabici (2021)

- Cannot know the positions and times of all sources; instead:
 - 1 Draw *random* positions and times from $s(\mathbf{x}', t')$
 - 2 Compute $\sum_n \mathcal{G}(\mathbf{x} - \mathbf{x}_n, t - t_n) s_p(p)$ for different realisations
 - 3 Compute intensity PDFs at different p :

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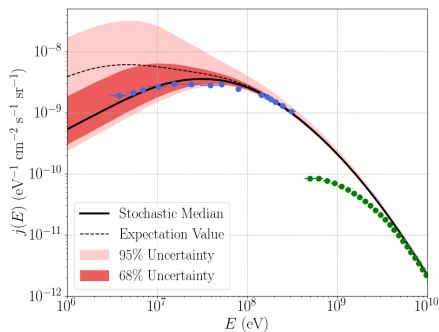


- At high energies (p_3):
Gaussian PDF
- At intermediate energies (p_2):
power law $\propto j^{-2.5}$,
second moment divergent
- At low energies (p_1):
power law $\propto j^{-2}$,
first moment divergent, too

How to quantify prediction
and its uncertainty?

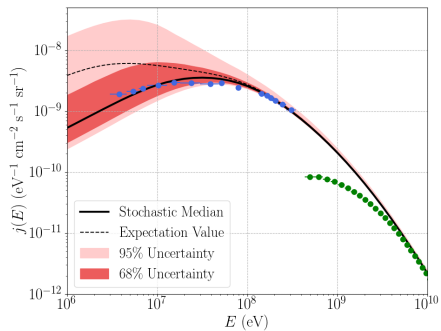
Results: protons

Phan, Schulze, Mertsch, Recchia, Gabici (2021)

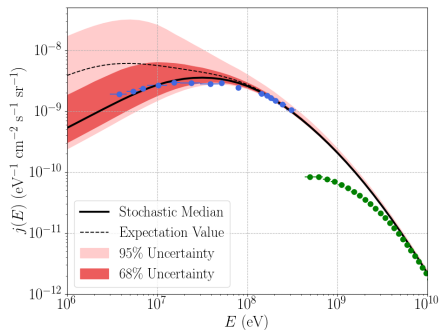


- Comparison with [Voyager 1](#) and [AMS-02](#) data
- [Voyager 1](#) data inside stochastic uncertainty band
- Expectation value far from median at low energies

Interpretation



Interpretation



$$p \gg p_*$$

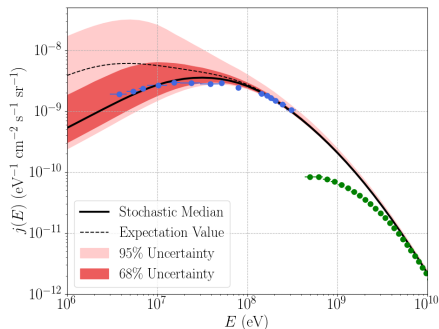
- Many sources contribute

$$\psi(p) \propto \frac{s_p(p)}{\kappa(p)}$$

- $s_p(p) \propto p^{-2.2}$, $\kappa(p) \propto p^{0.6}$

$$\Rightarrow j \propto E^{-2.8}$$

Interpretation



$p \ll p_*$

- Only a single source contributes

$$\lim_{p \ll p_*} \mathcal{G} \simeq \frac{1}{|\dot{p}|}$$

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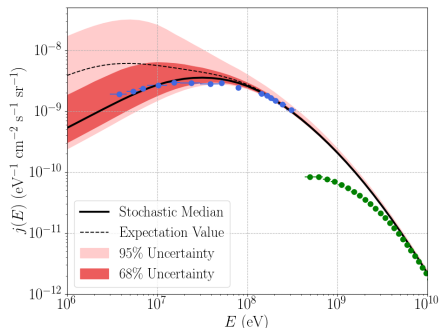
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$$\lambda_d = \sqrt{4\kappa(p)\tau(p)}$$

with $\tau(p) = p/|\dot{p}|$

- number of srcs. $\stackrel{!}{=} 1$

$$\mathcal{R}_{Tl}(p_*) \frac{\lambda_d(p_*)^3}{V_{\text{srcs}}} = 1$$

$$\Rightarrow E_* \sim 10 \text{ MeV}$$

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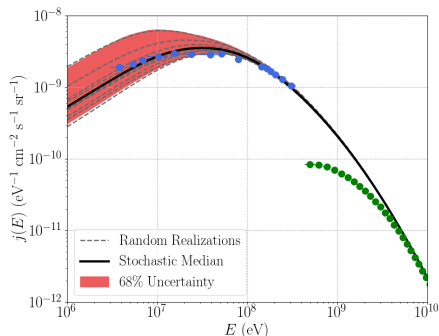
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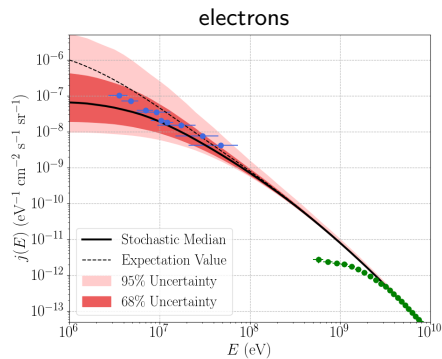
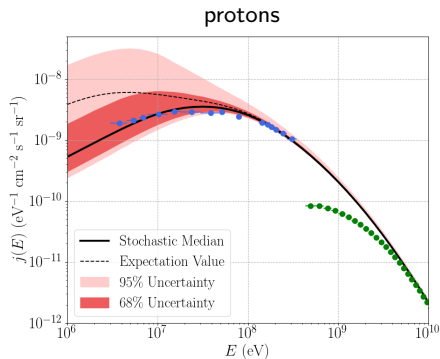
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Results: protons & electrons

Phan, Schulze, Mertsch, Recchia, Gabici (2021)



- Voyager 1 data inside uncertainty band
- Source discreteness effects important

→ No need for unmotivated break in $s_p(p)$!

